A <u>brief review</u>: In genus zero, $b = |\partial S|$.

Fix $f \in \mathrm{Aut}(\mathcal{C}(S))$. Fix α, β dual pants curves.

(1) $\exists f_\alpha \in \mathcal{MCG}(S)$ s.t. $f_\alpha \circ f \,\big|\, \{\alpha\} = \mathrm{Id}$.

(2) $\exists f_{lk} \in \mathcal{MCG}(S)$ s.t. $f_{lk} \circ f_\alpha \circ f \,\big|\, \{\alpha\} \cup lk(\alpha) = \mathrm{Id}$

(3) $\exists n \in \mathbb{Z}$ s.t. $T_\alpha^{n/2} \circ f_{lk} \circ f_\alpha \circ f \,\big|\, \{\alpha, \beta\} \cup lk(\alpha) = \mathrm{Id}$.

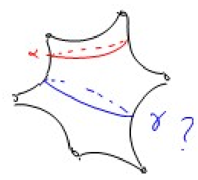
<u>So</u>: Define $\phi = T_\alpha^{n/2} \circ f_{lk} \circ f_\alpha \circ f$.

<u>Crawl in dual(α)</u>: $\phi \,|\, \mathrm{dual}(\alpha) = \mathrm{Id}$

<u>Crawl in $\mathcal{C}(S)$</u>: $\phi = \mathrm{Id}$. $\therefore f \in \mathrm{Aut}(\mathcal{C}(S))$ was the image of $(T_\alpha^{n/2} \circ f_{lk} \circ f_\alpha)^{-1}$ and we are done. //

<u>Detail regarding (1)</u>: We must show that α, $f(\alpha)$ have the same topological type!
[I.e. Why is $f(\alpha)$ a pants curve?]

Eg in $S = S_{0,6}$



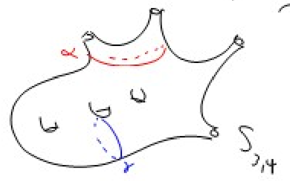
<u>However</u>:

<u>Claim</u>: If $S = S_{0,b}$ $(b \geq 6)$ then α is a pants curve iff $lk(\alpha)$ is large.

<u>Thus</u>: α pants $\Rightarrow lk(\alpha)$ large $\Rightarrow lk(f(\alpha))$ large $\Rightarrow f(\alpha)$ is pants.

This means that f_α exists in step (1) above

Life is more complicated in the presence of genus.



<u>Notice</u>: $lk(\alpha), lk(\gamma)$ both large <u>but</u> α, γ of different types.

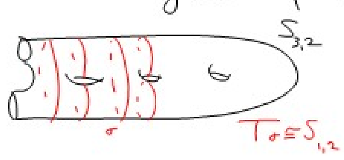
We need a combinatorial device to distinguish non-sep loops from pants curves.

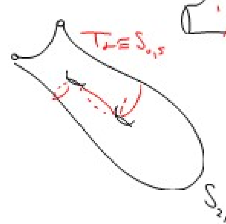
Suppose $\xi(S) \geq 3$. Let $\sigma \subseteq \mathcal{C}(S)$ be a simplex with (*) $|\sigma| = \xi(S) - 2$.

(*) $lk(\sigma)$ is large.

<u>Then</u>: $S_\sigma = S \setminus \dot{N}(\sigma)$ has exactly <u>one</u> non pants component, T_σ,

<u>Examples</u>:



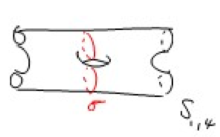


<u>and</u> $T_\sigma \cong S_{0,5}$ or $S_{1,2}$

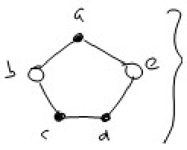
<u>Pf</u>: $3g - 3 + b = 2$ iff $g = 1, b = 0$ or $g = 0, b = 5$ [b/c $g > 0$, $b > 0$]

Also: T_σ is connected: If not $lk(\sigma)$ is small. //

<u>Non-example</u>:



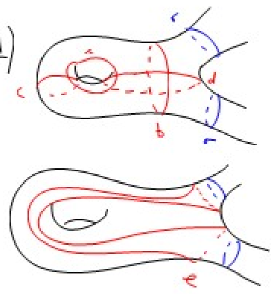
Suppose that $\{a, b, c, d, e\} = P$ is a pentagon in $lk(\sigma)$ where $|\sigma| = \xi(S) - 2$ and $lk(\sigma)$ large

<u>And</u>: 

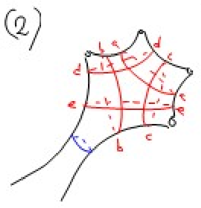
a, c, d have large link
b, e have small link

<u>Lemma</u>: <u>Then</u> either (1) a, c, d are nonsep
a dual to c, d

or (2) a, c, d are points curves
a dual to c, d.

<u>Pictures</u>: (1) 

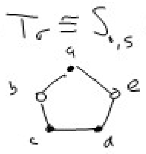
and the pentagon P in $\mathcal{C}(S_{1,2})$.

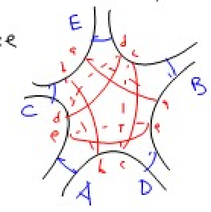
Picture (2) 
$P \subset \mathcal{C}(S_{0,6})$
Also, we learn that
$|\partial S| \geq 4$.

<u>Def</u>: If P is a <u>closed pentagon</u> and a, c are non-adjacent as above say a, c are <u>combinatorially dual</u> [$\Rightarrow$ dual + more]

<u>Pf of Lemma</u>: $\{a,b,c,d,e\} = P \subseteq \mathrm{lk}(\sigma)$ all as above, $\xi(S) \geq 3$. $T_\sigma \cong S_{0,5}$ or $S_{1,2}$.

If $T_\sigma = S_{1,2}$ then <u>Exercise</u> [Hints: 2 possible pf techniques (1) Copy pf of pentagon lemma (2) Use pentagon lemma in $\mathcal{C}(S_{0,5})$ and the exercise that $\mathcal{C}(S_{0,5}) \cong \mathcal{C}(S_{1,2})$: Think about hyperelliptic.]

If $T_\sigma \cong S_{0,5}$: we see 

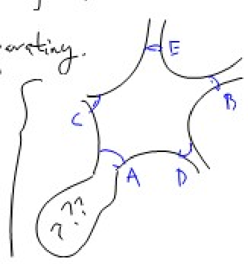


let $T_A, T_B, \ldots, T_E$ be the component of $S \setminus \mathring{T}_\sigma$ meeting $A, B, \ldots, E$ respectively.

<u>Case</u>: If $T_A = T_D$ then b is nonsep so has large link $\divideontimes$

So $T_A \neq T_D$. Similarly $T_A \neq T_B$, $T_A \neq T_E$

<u>Similarly</u>: $T_A \neq T_C$ (using e) <u>So</u> deduce that A is separating.

<u>We see</u>. 

<u>Will finish next time</u>