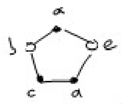
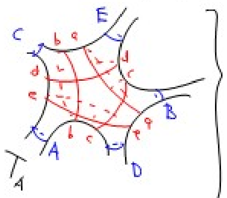
Recall: Proving Ivanov's Thm for $g \geq 1$.

Lemma from last time: 

○ small link
• large link

inside of $\mathcal{C}(T_\sigma)$ $\left[|\sigma| = \xi(S) - 2 \right]$



Defined $T_A, \cdots T_E$ to be the compts of $S \setminus T_\sigma$ meeting $A, \cdots, E$

If $T_A = T_C \Rightarrow e$ is nonsep $\divideontimes$

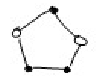
If $T_A = T_D \sim T_B \sim T_E \Rightarrow b$ is nonsep $\divideontimes$

Deduce that A is separating.

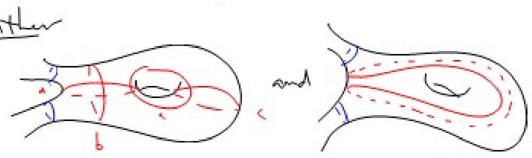
Similarly prove [Exercise] that C, D are separating.

Hence: a is separating. This and large link implies that a is a pants curve. [E, B are peripheral]

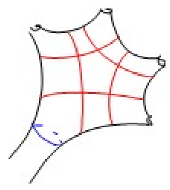
Similarly prove [Exercise] C, D are peripheral. //

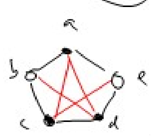
Picture:  implies the topology is

Either

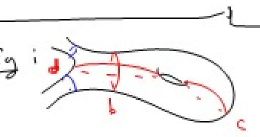


or



Notice: 

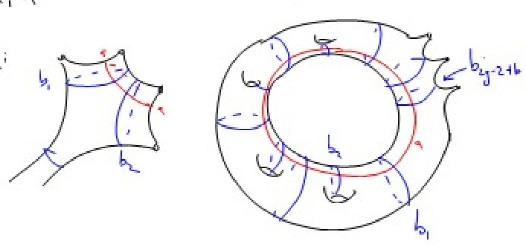
The red arcs denote duality of non adjacent vertices in the $S_{0,4} = T_\sigma$ cut along the curve in between.

Eg: 

b, d are dual in $(T_\sigma)_c = T_\sigma$ cut along c.

Def (Behrstock-Margalit) **Bracelet**.

Suppose $\alpha \in \xi(S)$ has large link and $\{b_1, \ldots b_n\}$ is a simplex s.t. $\forall i$ a, b_i are combin. dual,

Picture: 

Lemma: The maximal bracelet about a $\left\{ \begin{matrix} \text{pants curve} \\ \text{non-sep loop} \end{matrix} \right\}$ has size $\left\{ \begin{matrix} 2 \\ 2g-2+b \end{matrix} \right\}$.

Pf: [Exercise]

Consequence: When $2g-2+b \geq 3$ we can distinguish (combinatorially) nonsep from pants curves.

$2g-2+b = 2 \Rightarrow S = \cancel{S_{2}}, S_{1,2}, \cancel{S_{0,4}}$

$2g-2+b = 1 \Rightarrow S = \cancel{S_{1,1}}, \cancel{S_{0,3}}$

Of these cases the crossed out ones are uninteresting.

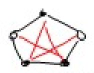
$S_{0,3}$ has no curve complex
All loops non sep in $S_{1,1}$
" " sep in $S_{0,4}$
S_2 contains no pants curves.

we are left with $S_{1,2}$.

Lemma [Luo] Define $\mathcal{C}^c(S_{1,2})$ to be the curve complex of $S_{1,2}$ with non-sep vertices colored black and sep vertices colored white.

$$\mathrm{Aut}(\mathcal{C}^c(S_{1,2})) \cong \mathrm{MCG}(S_{1,2})/\langle\tau\rangle$$

Rmks: ① $\mathrm{Aut}(\mathcal{C}(S_{1,2})) = \mathrm{Aut}(\mathcal{C}(S_{0,5})) = \mathrm{MCG}(S_{0,5})$
because $\mathcal{C}(S_{1,2}) \cong \mathcal{C}(S_{0,5})$.
② $\mathrm{MCG}(S_{1,2}) \underset{5}{\leq} \mathrm{MCG}(S_{0,5})$ by Luo's Lemma.

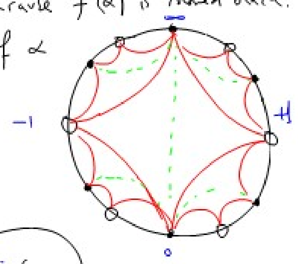
Pf: (of Lemma) All pentagons are colored 
[By Pentagon lemma for $S_{1,2}$] (Exercise)

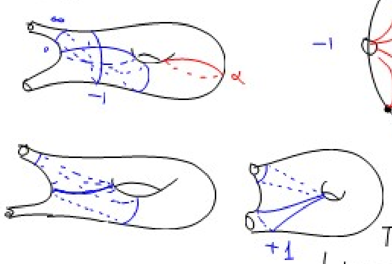
• nonsep
∘ sep

Fix $f \in \mathrm{Aut}(\mathcal{C}^c)$, α, β comb. dual.

(1) $\exists f_\alpha$ is automatic because $f(\alpha)$ is colored black.

(2) $\exists f_{1k}$: Consider the link of α





This is $+1$
because $i(0,1) = i(\infty,1) = 2$
$i(1,-1) = 4$

Also checked $+1$ is pants curve.

The duality given by pentagons gives a square tessellation of $\mathcal{F}^c(T_\alpha)$

Ex: Check that $\mathrm{Aut}(\mathcal{F}^c) < \mathrm{Aut}(\mathcal{F}) = PGL(2,\mathbb{Z})$

Ex: Compute $\mathrm{Aut}(\mathcal{F}^c)$ (Hint: It is index 3 in $PGL(2,\mathbb{Z})$)

By the above: f_{1k} exists and [check] after composing with hyperelliptic f_{1k} preserves α^+ and α^-. So we may glue f_{1k} along α to get a homeo of $S_{1,2}$.

(3) $\exists n$ s.t. $T_\alpha^n \circ f_{1k} \circ f_\alpha \circ f(\beta) = \beta$ is easy.

Next time crawling / end and A.