Exercise 1.1. Google a bit of the history of knot theory. Names that will appear in this course include Gauss, Lord Kelvin, Tait, C.N. Little, Haseman, Reidemeister, Alexander, Conway, Dowker.

Exercise 1.2. Suppose that D is an oriented diagram of a knot (or link) and $-D$ is the diagram with opposite orientation. Prove that $w(-D) = w(D)$.

Exercise 1.3. Suppose that D is an oriented diagram and $\overline{D}$ is the mirror-image diagram. Prove that $w(\overline{D}) = -w(D)$.

Exercise 1.4. Suppose that $D = \bigsqcup C_i$ is a diagram. Prove that $\mathrm{lk}(C_i, C_j)$ is an integer.

Exercise 1.5. Prove the easy direction of Reidemeister's theorem.

Exercise 1.6. Show that the R_∞ move can be obtained as via a sequence of the standard four moves.

Exercise 1.7. Show that the figure eight is isotopic to its mirror image. (Use a piece of string!) Now draw a sequence of Reidemeister moves to prove that the two knots are isotopic. (Hint: Exercise 1.6 may be useful.)

Exercise 1.8. The figure eight has two orientations. Are these isotopic? If so, provide a sequence of Reidemeister moves.

Exercise 1.9. Provide a short proof that the unlink and the Hopf link are not isotopic. Think about how you would prove that the unlink and the Whitehead link are not isotopic.

Exercise 1.10. As done in the notes for the trefoil and the figure eight, find non-trivial colorings of the Whitehead link. Careful: you cannot divide by two in the ring $\mathbb{Z}_{2m}$. (That is, when the modulus is even.)