These exercises are mainly taken from the second week's lectures. Please do let me know if any of the problems are unclear or have typos.

Exercise 2.1. Show that the standard diagram of the Borromean rings cannot be 3–colored.

Exercise 2.2. Draw a shadow where at least two of the regions are not disks.

Exercise 2.3. Suppose that the diagram D has shadow S. Suppose that S has a at least two regions which are not disks. Show that D is splittable.

For the next several exercises, A_+ is the matrix of crossing equations and A is the matrix obtained by deleting a row and column from A_+.

Exercise 2.4. Compute A_+ for the twist knots, T_k.

Exercise 2.5. Show that the absolute value $|\det(A)|$ is independent of the choice of row and column deleted from A^+.

Exercise 2.6. [Harder] Show that the Smith normal form of A is independent of the choice of row and column deleted from A^+.

Exercise 2.7. In our use of Cramer's rule (to find possible solutions to the coloring equations) the choice of b was not specified. Setting $b = (1, 0, \ldots, 0)$ we find that $x_k = (-1)^{k+1}\mathrm{Minor}_{1,k}(A)$ and also that $\det(A) = A^1 \cdot x$ where A^1 is the first row of A. Use this to find a coloring modulo $\det(K)$ of the knot 6_3.

Exercise 2.8. Check that if a diagram is *alternating* (every overcrossing arc goes over exactly one crossing) then the variables may be ordered so that the matrix A^+ has twos along the diagonal.

Exercise 2.9. Do Exercise 11 on Sanderson's example sheet:

`http://www.warwick.ac.uk/~maaac/examples2.html.`

Here a quadrilateral decomposition is the planar graph *dual* to the shadow.