

These exercises are mainly taken from the third week's lectures. Please do let me know if any of the problems are unclear or have typos.

Exercise 3.1. Compute the determinant of the 5_1 knot (the cinefoil) and the twist knots. Notice that the first two twist knots are the trefoil and the figure eight.

Exercise 3.2. [Harder] Recall that A_+ is the matrix of crossing equations and A is a matrix obtained by deleting any one row and column from A_+ . Prove that the Smith Normal Form of A is independent of our choice of row and column. (Hint: it will be helpful to choose the signs of the rows so that the entries of any single column sum to zero.)

Exercise 3.3. Let $T(2, 4)$ be the $(2, 4)$ -torus link. Let W be the Whitehead link. Show that $\det(T(2, 4)) = 4$ while $\det(W) = 8$.

Exercise 3.4. Show that $P = P(-2, 3, 5)$ has determinant equal to one.

Exercise 3.5. [Harder] Compute the coloring group of the pretzel link $P = P(p, q, r)$. Determine which triples (p, q, r) give a trivial group.

Exercise 3.6. Prove that the coloring group is an isotopy invariant. To do this show that if two diagrams D and D' differ by a single Reidemeister move then $\text{Col}(D)$ and $\text{Col}(D')$ are isomorphic.

Exercise 3.7. Reproduce the direct computation of $\text{Col}(L)$, where L is the 12-crossing two-component "boundary link" with determinant zero, as performed in class.