These exercises are mainly taken from the forth week's lectures. Please do let me know if any of the problems are unclear or have typos.

Exercise 4.1. Check that 8_2 and 8_{17} have isomorphic coloring groups.

Exercise 4.2. If you haven't already done so, check that the figure eight knot (4_1) is achiral.

Exercise 4.3. Suppose that $P = P(p_1, p_2, \ldots, p_n)$ is a pretzel knot. Show that if one of the p_i is even then P is invertible. [Hard] Are all pretzel knots invertible?

Exercise 4.4 (Harder)**.** Verify the missing step in constructing codes: every crossing receives one odd and one even number.

Exercise 4.5. The knots 6_1, 6_2, 6_3, have codes

$$[4, 8, 12, 10, 2, 6] \quad [4, 8, 10, 12, 2, 6] \quad [4, 8, 10, 2, 12, 6]$$

respectively. Draw these and check that they are isotopic to the standard diagrams.

Exercise 4.6. Compute the codes for the granny and reef knots, shown in Figure 19.

Exercise 4.7. Prove that there are at most $2^n \cdot n!$ knots, up to isotopy, with n or fewer crossings. [Hard] Can you also give an exponential lower bound?

Exercise 4.8. Compute the Alexander polynomial for the trefoil knot by first computing the matrix of crossing equations. You should find that $\Delta_T(t) = t - 1 + t^{-1}$, up to multiplication by units in the ring $\mathbb{Z}[t, t^{-1}]$.

Exercise 4.9. Show that $w(R)$, the winding number of the oriented shadow around the region R, is well defined. (Hint: this is very similar to the proof, given in class, that the parity $e(R)$ is well-defined.) Show that if R, R' are adjacent then $w(R) = w(R') \pm 1$. Show that $w(R) = e(R) \mod 2$ where $e(R)$ is the parity of the region R.