

These exercises are mainly taken from the fifth week's lectures. Please let me know if any of the problems are unclear or have typos.

Exercise 5.1. Find a reduced diagram for the unknot with three distinct arcs at each crossing (zero crossings not allowed!). Now calculate the Alexander polynomial of the unknot.

Exercise 5.2. Compute the Alexander polynomial of the left and right Hopf links.

Exercise 5.3. A knot or link is *prime* if it does not decompose as a connect sum. Compute the Alexander polynomial for all prime knots up to six crossings by finding the matrix of crossing equations. (A computer algebra package will be useful for taking determinants.) Compute the Alexander polynomial for all prime links up to five crossings. What patterns do you find?

Exercise 5.4. Demonstrate the following properties of connect sum:

- $K \# U \cong K$
- $K \# L \cong L \# K$
- $(K \# L) \# M \cong K \# (L \# M)$.

Exercise 5.5. Which knots up to seven crossings are two-bridge? (It is easier by far to look for 4-plat position and then use the theorem proved in class.)