These exercises are mainly taken from the seventh week's lectures. Please let me know if any of the problems are unclear or have typos.

Exercise 7.1. Show that the number of Seifert circles of a diagram is not a knot invariant.

Exercise 7.2. Show that for any knot diagram the number of Seifert circles and the writhe have opposite parity.

Exercise 7.3. Provide the details the lemma claimed in class: If D is a diagram, and D' is the diagram after a conflict has been resolved, then D' has exactly one less disagreement than D.

Exercise 7.4. Redo the example done in class – isotope the $(2,4)$–torus link to be a braid closure.

Exercise 7.5. [Harder] Show that in addition to the R_2 and R_3 moves the Kauffman bracket is also invariant under the R_∞ move. Reviewing your answer to Exercise 1.6 may be helpful. (In fact, this justifies adding R_∞ to the definition of regular isotopy.)

Exercise 7.6. Compute the Kauffman bracket of the $(2,p)$–torus links. [Harder] Do the same for twist knots.

Exercise 7.7. Suppose that D, E are diagrams. Use the state sum formulation of the Kauffman bracket to show that

- $\langle D \amalg E \rangle = \langle D \rangle \langle E \rangle \cdot (-A^2 - A^{-2})$
- $\langle D \# E \rangle = \langle D \rangle \langle E \rangle$.

Exercise 7.8. Prove that the writhe of a diagram is a regular isotopy invariant.

Exercise 7.9. Verify that the right and left trefoils are not isotopic by computing their Kauffman polynomials.

Exercise 7.10. Prove that the Kauffman polynomial of an amphichiral knot is symmetric. Check this for the figure eight knot.