These exercises are mainly taken from the ninth week's lectures. Please let me know if any of the problems are unclear or have typos.

Exercise 9.1. Suppose that T is an arbitrary tangle. Suppose that p, q are integral tangles (ie twist boxes).

- Show that $p + q = q + p$.
- Show that $T + 0 = 0 + T = T$.
- Show that $p + \infty = \infty + p = \infty$.
- Show that $1/1/T = T$.

Here equality denotes tangle isotopy.

Exercise 9.2. [Hard] Find tangles S, T so that $S + T \neq T + S$. (ie they are not tangle isotopic.)

Exercise 9.3. Suppose that $T = 0$, $S = (-3).0$, and $R = 1$. Show that:

- $N(S + T) = N(1)$
- $N(S + R) = N(2)$
- $N(S + R + R) = N(3, 1, 1)$
- $N(S + R + R + R) = N(-1, -1, -1, -1, -1)$

Exercise 9.4. Show that:

- $N(1)$ is the unknot.
- $N(2)$ is the Hopf link.
- $N(2, 1, 1)$ is the figure eight.
- $N(1, 1, 1, 1, 1)$ is the Whitehead link.
- $N(1, 1, 1, 2, 1)$ is the 6_2 knot.

Exercise 9.5. Show that the mirror of a numerator satisfies

$$m(N(a_1, a_2, ..., a_n)) = N(-a_1, -a_2, ..., -a_n).$$

Exercise 9.6. [Hard] Every four-plat has an alternating diagram.

Exercise 9.7. [Harder] Give an upper bound on $g(K)$ where K is a four-plat in standard position.