Lecture 13

Q: Why require Area$(q)=1$?

A: In order to fix a scale.

Length: Fix q. Pick $\alpha \in \mathcal{C}_0(S)$

Define: $\ell^2_q(\alpha) =$ length of α^* in $S(q)$.

Define: $\ell^v_q(\alpha) = \int_{\alpha^*} |dy| =$ vertical length

$\ell^h_q(\alpha) = \int_{\alpha^*} |dx| =$ horizontal length

Def: Say α is vertical (horizontal) if $\ell^h = 0$ $(\ell^v = 0)$

Def: $$\ell^\infty_q(\alpha) = \int_{\alpha^*} \max\{|dx|,|dy|\} = \sum_{\sigma \subseteq \alpha^*} \max\{\ell^h(\sigma), \ell^v(\sigma)\}$$
$\sigma \subseteq \alpha^*$ is a maximal $\mathbb{E}^2$ segment.

Exercise: Show that

$$\ell^v + \ell^h, \quad \ell^2, \quad \ell^\infty, \quad \underset{[\text{fake } \ell^\infty]}{\max\{\ell^h, \ell^v\}}$$

are all comparable [up to multip. error]

Combing "lines" $S = S_{g,n}$ $R_0 = R_0(S)$ suff. large.

$\forall R \geq R_0$ define

$$L(q,R) = \{\alpha \in \mathcal{C}_0(S) \mid \ell^2_q(\alpha) \leq R\}$$

[NB: Possible to have $\delta, \varepsilon \in L(q,R)$ s.t. $i(\delta,\varepsilon)$ very large]

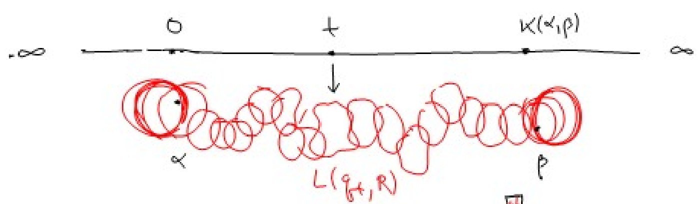
Let $q_t = q_t^{\alpha\beta}$.

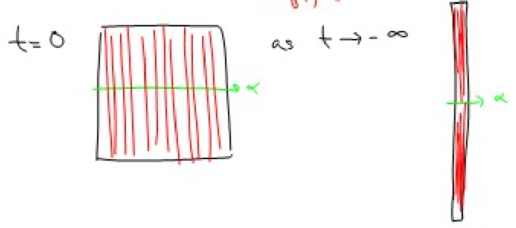
Let $$L^{\alpha\beta}[s,t] = \bigcup_{r \in [s,t]} L(q_r^{\alpha\beta}, R)$$

$$L^{\alpha\beta}(-\infty) = N_{\underline{1}}(\alpha) = \{\delta \in \mathcal{C}_0(S) \mid i(\alpha,\delta) = 0\}$$

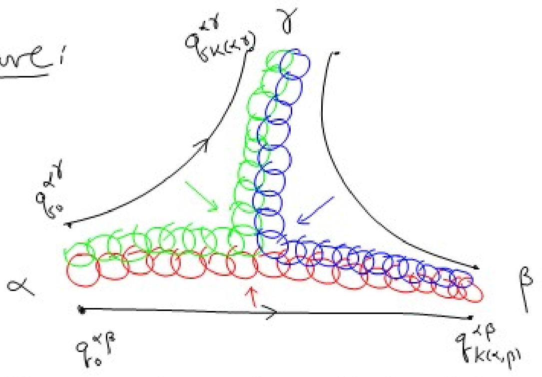
$$L^{\alpha\beta}(\infty) = N_{\underline{1}}(\beta)$$

$$\Lambda^{\alpha\beta} = L^{\alpha\beta}(-\infty) \cup L^{\alpha\beta}(-\infty,\infty) \cup L^{\alpha\beta}(\infty)$$





<u>Picture:</u>



<u>Hope for</u> ① $\mathrm{diam}_{\mathcal{C}}(L(q, R))$ bounded.

② $\Lambda^{\alpha\beta}$ is coarsely connected
[$\forall t\ \exists \varepsilon$ s.t. $L(q_t)$ is close to $L(q_{t+\varepsilon})$]

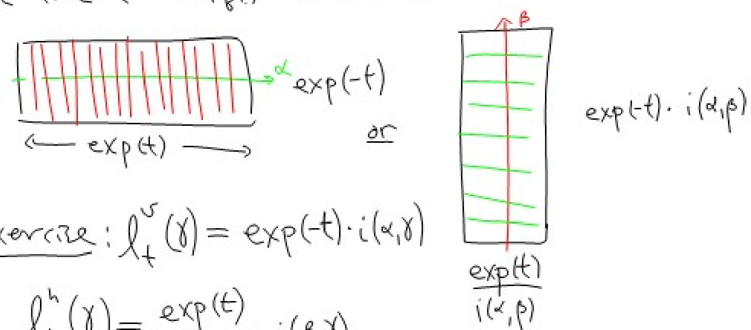
③ $\Lambda^{\alpha\beta\gamma}$ is <u>slim</u> ie has a center, looks like a tripod.

<u>Balance time:</u> Say $\gamma \in \mathcal{C}(S)$ is <u>balanced</u> in q if $\ell^h_q(\gamma) = \ell^v_q(\gamma)$

For $q = q^{\alpha\beta}_t$ Call $t = t_{\alpha\beta}(\gamma)$ the balance time of γ along $q^{\alpha\beta}_t$

<u>Existance and uniqueness of</u> $t_{\alpha\beta}(\gamma)$

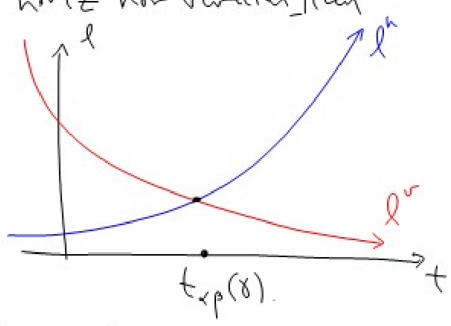
At time t $S(q_t)$ looks like



<u>Exercise:</u> $\ell^v_t(\gamma) = \exp(-t)\cdot i(\alpha,\gamma)$

$$\ell^h_t(\gamma) = \frac{\exp(t)}{i(\alpha,\beta)}\cdot i(\beta,\gamma)$$

Thus if if $i(\alpha,\gamma), i(\beta,\gamma) \neq 0$ [γ neither horiz nor vertical] then



<u>Important exercise</u>

$$t_{\alpha\beta}(\gamma) = \frac{1}{2}\left(K(\alpha,\beta) + K(\alpha,\gamma) - K(\beta,\gamma)\right)$$ ← compare to tripod def of δ-hyp

[Recall $K(\alpha,\beta) = \log(i(\alpha,\beta))$]

① Not symmetric in α,β [b/c parametrization of q_t was not symmetric.]

② Instead symmetric is β,γ.
That is balance time for γ on $q^{\alpha\beta}$ agrees with " " " β on $q^{\alpha\gamma}$: $t_{\alpha\beta}(\gamma) = t_{\alpha\gamma}(\beta)$