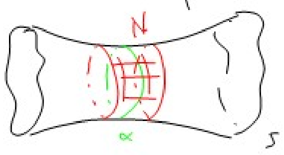
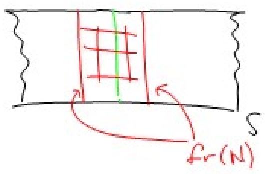
Lecture II Saul Schleimer

(I) Cutting: Suppose $\alpha \subset S$

Let $N(\alpha)$ be a closed regular neighborhood, equipped with a product structure $\alpha \times I$



Def $fr(N)$ = frontier of $N = \overline{\partial N \setminus \partial S}$



Def: $\mathring{N} = N \setminus fr(N)$

Def: $S \setminus\setminus \alpha = S$ cut along $\alpha = S \setminus \mathring{N}(\alpha)$.

Correct def of iness.

Def: $\alpha \subset S$ is inessential

iff

- α is separating
- $\underbrace{S \setminus\setminus \alpha}_{S \text{ cut along } \alpha}$ has a disk component

iff

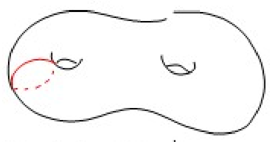
- $\exists$ component $X \subset S \setminus \alpha$ s.t. $\overline{X}$ is a disk (closure taken in S).

Exercise: Give definition of peripheral curve.

Exercise: $f \sim g$ is an equivalence rel.

Exercise: $MCG(S)$ is countable [Hard]

Exercise: Draw an interesting curve on a surface. Not

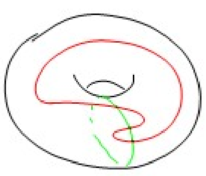


(II) Geometric intersection number.

Def: $\alpha, \beta \in \mathcal{C}^\circ(S)$ then

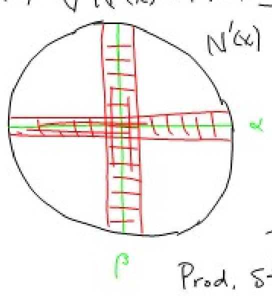
$$i(\alpha,\beta) = \min\{|\alpha' \cap \beta'| : \alpha' \sim \alpha, \beta' \sim \beta\}$$

Exercise $= \min\{|\alpha' \cap \beta| : \alpha' \sim \alpha\}$



Def: $\alpha, \beta \subset S$ are transverse at $x \in \alpha \cap \beta$

if $\exists N(x)\ \forall N'(x) \subset N(x)\ \exists N(\alpha), N(\beta)$ so that



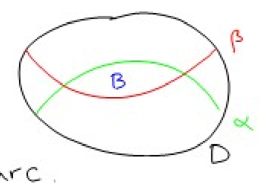
$N(\alpha) \cap N'$, $N(\beta) \cap N'$ are rectangles

$N(\alpha) \cap N(\beta) \cap N'$ is a square

Prod. str. rotate by $90°$

Ex: If α, β transverse then $|\alpha \cap \beta| < \infty$.

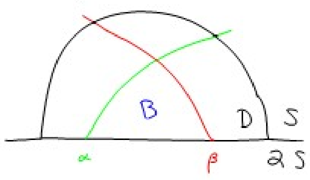
<u>Def</u>: α, β <u>share a bigon</u> if $\exists$ disk $D \subset S$ s.t.



$\alpha \cap D$, $\beta \cap D$ is each a single arc.

$|\alpha \cap \beta \cap D| = 2$, both points transverse.

<u>Def</u>: α, β share a <u>triangle on ∂S</u> if



Notice that if you double along ∂S, triangles $\Rightarrow$ bigons

<u>Bigon Criterion</u>: Suppose α, β transverse. α, β share no bigons or triangles iff $|\alpha \cap \beta| = i(\alpha, \beta)$.

<u>Reading Exercise</u>: Find a proof of this. Read it.

<u>Note</u>: $i(\alpha, \beta) = 0$ iff α, β have disjoint reps.

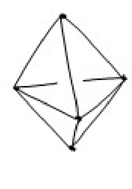
Say $\Delta \subset \mathcal{AC}^\circ(S)$ is a <u>multicurve</u> if $\forall \alpha, \beta \in \Delta$, $i(\alpha, \beta) = 0$.

<u>Exercise</u>: Δ can be realized disjointly (all at same time)

III Simplicial Complexes.

Suppose K° is a set (of <u>vertices</u>)

Then $K \subset \mathcal{P}(K^\circ)$ is a <u>simp complex</u> if $\emptyset \notin K$ and $\forall \tau \overset{\neq \emptyset}{\subseteq} \sigma \in K$, $\tau \in K$

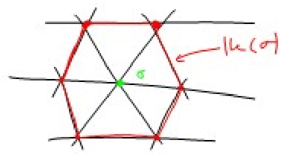


if $\sigma \in K$

$\mathrm{lk}(\sigma) = \mathrm{link}(\sigma)$

$= \{ \tau \in K \mid \tau \cap \sigma = \emptyset, \tau \cup \sigma \in K \}$

Delete to get star(σ).



Note if σ is a vertex (singleton) then $\mathrm{star}(\sigma) = \mathrm{Cone}(\sigma, \mathrm{lk}(\sigma))$.

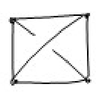
<u>Def</u>: K, L simp complexes, $K^\circ \cap L^\circ = \emptyset$

$$K \vee L = K \cup L \cup \{ \sigma \cup \tau \mid \sigma \in K, \tau \in L \}$$

<u>Exercise</u>: $S^n \vee S^m \cong S^{n+m+1}$

$\uparrow$ n-dim'l sphere.

<u>Define</u>: $\mathcal{AC}(S)$ with vertex set $\mathcal{AC}^\circ$ and multicurves give simplices.

$I \vee I =$ 

$=$ tetrahedron.

Ex