Lecture XI

Recall: $g \subseteq \mathcal{C}(S)$ is a *tight* geodesic

if $g = \{\sigma_i\}_{i \in K}$ (where $K \subset \mathbb{Z}$ is either an interval, a ray or all of $\mathbb{Z}$)

has ⊛ $\forall \alpha_i \in \sigma_i$ the sequence $\{\alpha_i\}$ is a geodesic

(⁎⁎) $\forall i$ σ_i is a simplex

(⁎⁎⁎) $\forall i$, $\sigma_i = \partial_S X(\sigma_{i-1}, \sigma_{i+1})$

Lemma: $\forall \alpha, \beta \in \mathcal{C}(S)$ $\exists$ a tight geodesic connecting α to β. //

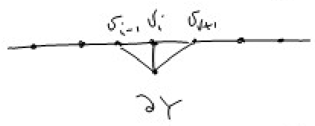
Also: We assume that all tight geodesics $\hat{g}$ come with $I(g)$, $T(g)$

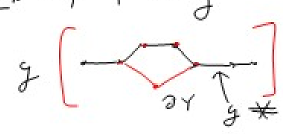
initial and *terminal* markings

Foot prints: Suppose $Y \subseteq S$ is essential. Suppose $g \subseteq \mathcal{C}(S)$ is tight. Define

$$\phi_g(Y) = \{ v_i \in g \mid v_i \text{ misses } Y \}$$

$$= \text{\underline{footprint} of } Y \text{ on } g$$





Lemma: If g is tight then $\phi_g(Y)$ is $0, 1, 2$ or 3 *consecutive* vertices of g

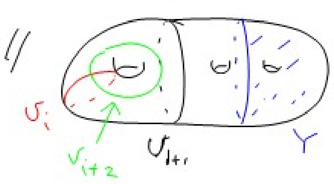
Pf: Suppose $v_j = \max \phi_g(Y)$

$v_i = \min \phi_g(Y)$.

Because g geodesic $j - i \leq 2$.

If $j = i+2$ then $v_{i+1} = \partial_S X(v_i, v_{i+2})$

Since $v_i \cup v_{i+2}$ misses Y the same holds for v_{i+1}. //



Hierarchies

All definitions also hold in subsurfaces.

(A) $\sigma \subseteq \mathcal{C}(S)$ is a simplex, and $Y \subseteq S \setminus \sigma$ is a connected component.

Call Y a *component domain* for σ.

(B) If $g \subseteq \mathcal{C}(Y)$ is tight then write $D(g) = Y$ (domain of g)

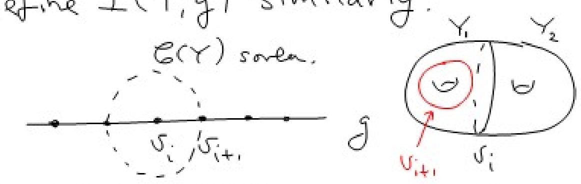
(C) If μ is a marking $\mu = \{(\alpha, \beta)\}$

Define $\mu|_Y = \begin{cases} \{(\alpha,\beta) \mid \pi_Y(\alpha) \neq \emptyset\} \\ \pi_Y(\mu) \text{ if } Y \cong A^2. \end{cases}$

Ⓓ Suppose g is tight, $Y \subseteq S$ is a component domain for $v_j \in g$

$$T(Y,g) = \begin{cases} v_{j+1}|_Y & \text{if } v_j \text{ not last} \\ T(g)|_Y & \text{if } v_j \text{ is last.} \end{cases}$$

Define $I(Y,g)$ similarly.



So $T(Y_1,g) = v_{i+1}$ and $T(Y_2,g) = \emptyset$.

Ⓔ Subordinate

If Y is a compt domain for $g = \{v_i\}$ and $\left\{\begin{matrix} T(Y,g) \\ I(Y,g) \end{matrix}\right\} \neq \emptyset$ we say Y is directly $\left\{\begin{matrix} \text{forwards} \\ \text{backwards} \end{matrix}\right\}$ subord. to g, writing $\left\{\begin{matrix} Y \overset{d}{\searrow} g \\ g \overset{d}{\swarrow} Y \end{matrix}\right\}$

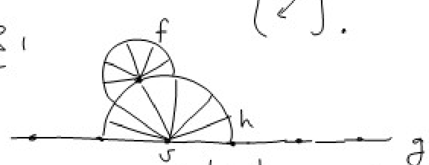
Def: If $k \subseteq \mathcal{C}(Y)$ is ~~tight~~, $Y \subseteq S$, $g \subseteq \mathcal{C}(S)$ is tight,

then write $\left\{\begin{matrix} k \overset{d}{\searrow} g \\ g \overset{d}{\swarrow} k \end{matrix}\right\}$ if $\left\{\begin{matrix} Y \overset{d}{\searrow} g \\ g \overset{d}{\swarrow} Y \end{matrix}\right\}$ and

$$\left\{\begin{matrix} T(Y,g) = T(k) \\ I(Y,g) = I(k) \end{matrix}\right\}.$$

Ⓕ Finally: Let $\left\{\begin{matrix} f \searrow g \\ g \swarrow f \end{matrix}\right\}$ denote the transitive closure of $\left\{\begin{matrix} \overset{d}{\searrow} \\ \overset{d}{\swarrow} \end{matrix}\right\}$.

Picture:



$f \overset{d}{\searrow} h \overset{d}{\searrow} g \Rightarrow f \searrow g$

Also $\underset{\overset{\|}{S}}{D(g)} \supseteq D(h) \supset D(f)$.

Def: $H = \{f\}$ a collection of tight geodesics is a hierarchy

① $\exists\, g_H \in H$ the main geodesic with $D(g_H) = S$

② If $b, f \in H$ and $Y \subseteq S$ has $b \overset{d}{\swarrow} Y \overset{d}{\searrow} f$ $\exists$ unique $k \in H$ s.t.

$$D(k) = Y, \left\{\begin{matrix} T(k) = T(Y,f) \\ I(k) = I(Y,b) \end{matrix}\right\}$$

③ $\forall\, k \in H \setminus \{g_H\}$ $\exists\, b, f \in H$ s.t. $b \overset{d}{\swarrow} k \overset{d}{\searrow} f$.