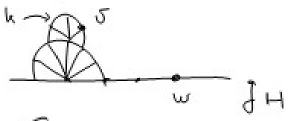
Lecture XIV

Suppose $\{v_0, v_1, \dots, v_N\} = k \in H$

write $v_i < v_j$ if $i < j$

and $I(k) < v_i < T(k)$ $\forall i$
$\left[\begin{array}{l}\text{if } v_i \neq I(k), \\ \quad v_i \neq T(k)\end{array}\right]$

So let $\{I(k), v_0, v_1, \dots, v_N, T(k)\}$ be the set of <u>positions</u> of k and call (k, v) a <u>pointed geodesic</u> if v is a position.



<u>Define</u> $\hat{\emptyset}_h(k,v) = \begin{cases} v, & \text{if } k = h \\ \emptyset_h(k), & \text{if } D(k) \subseteq D(h) \end{cases}$

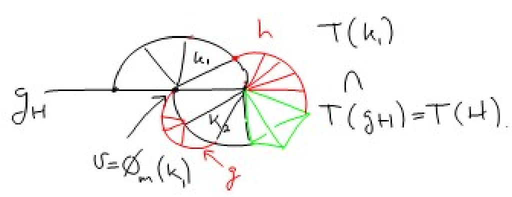
<u>Define</u>: $(k,v) <_p (h,w)$ if $\exists m \in H$
s.t. $\max \hat{\emptyset}_m(k,v) < \min \hat{\emptyset}_m(h,w)$

<u>Exercise</u>: $(k,v) <_p (h,w)$ iff exactly one of the following

(i) $k <_t h$
(ii) $k = h$ and $v < w$
(iii) $k \gg h$ and $\max \emptyset_h(k) < w$
(iv) $k \ll h$ and $v < \min \emptyset_k(h)$

<u>Lemma</u> (Time order)
(i) if $h, h' \in H$ are nested (i.e. $D(h), D(h')$ are)
then h, h' are <u>not</u> time ordered.
(ii) if h, h' overlap then h, h' <u>are</u> time ordered.
(iii) If $b \leqq k \geqq f$ then $b = f$, $b \gg f$, $b \ll f$, or $b <_t f$
(iv) If $h \geqq k_1 <_t k_2 \leqq g$ then
$h <_t k_2$ and $k_1 <_t g$

(v, vi) $<_t$, $<_p$ are strict partial orders.



<u>Pf</u> of (1) $\forall m \in H$ either $\emptyset_m(h), \emptyset_m(h') = \emptyset$
or $\emptyset_m(h) \cap \emptyset_m(h') \neq \emptyset$.
[in fact $\emptyset(h), \emptyset(h')$ nested]

<u>Pf of (2)</u>, Pick $m \in H$, minimizing $\xi(m)$,
s.t. (*) $D(h) \cup D(h') \subseteq D(m)$
(**) either $h \gg m$ or $h' \gg m$

[Exercise: m exists]

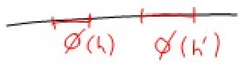
Step 1: If $D(h) = D(m)$ then $D(h') \subseteq D(h)$ $\cancel{\ast}$

Similarly and thus $D(h), D(h') \subsetneq D(m)$.

Coro [last time] $\emptyset_m(h), \emptyset_m(h') \neq \emptyset$.

if $\emptyset(h) \cap \emptyset(h') = \emptyset$ we are done.

Step 2



Suppose $h \searrow m$

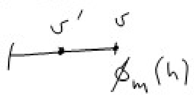
Case A: $\max \emptyset_m(h) = v \in \emptyset_m(h')$

Fix $m' \in H$ s.t. $h \trianglerighteq m' \overset{d}{\searrow} m$.

So $D(m')$ is a compt domain of (m, v). Since $v \in \emptyset_m(h')$, v misses $D(h')$. So since $D(h) \cap D(h') \neq \emptyset$ and $D(h) \subseteq D(m') \implies D(h') \subseteq D(m')$

$\implies$ Since $\xi(m') < \xi(m)$, $\cancel{\ast}$.

Case B: $\max \emptyset_m(h') = v' < v$



[$v = \max \emptyset_m(h)$]

$\implies$ v cuts $D(h') \Rightarrow m \in \Sigma_1^+(h')$

$\implies$ By lemma (Sub Int 2) $h' \searrow m$

So go back to case A, switching h, h'. //②

Lemma (Sub Int 3) H hier, Y a compt domain of (k, v). If $f \in \Sigma_{1H}^+(Y)$ then either $Y \searrow f$ or $D(f) = Y$.

Pf Idea: Fix (Y, k, v) a <u>triple</u>

[i.e Y is compt dom. of (k, v)]

Define: $N(Y, k, v) =$

$$\{ (Y', k', v') \mid Y \subseteq Y', (k, v) <_p (k', v') \}$$

Exercise: For any $(Y', k', v') \in N(Y, k, v)$ have $N(Y', k', v') \subsetneq N(Y, k, v)$

Exercise $N(Y, k, v)$ is finite

Idea: Induct on $N(Y, k, v)$.

Find (Y', k', v') later

So $\exists h'$ with $Y' \overset{d}{\searrow} h'$