Exercise 1.1. Google a bit of the history of knot theory. Names that will appear in this course include Gauss, Lord Kelvin, Tait, C.N. Little, Haseman, Reidemeister, Alexander, Conway, Dowker.

Exercise 1.2. Suppose that D is an oriented diagram of a knot (or link) and $-D$ is the diagram with opposite orientation. Prove that $w(-D) = w(D)$, where $w(D)$ is the write of the diagram D.

Exercise 1.3. Suppose that D is an oriented diagram and $\overline{D}$ is the mirror-image diagram. Prove that $w(\overline{D}) = -w(D)$.

Exercise 1.4. Suppose that $D = \bigsqcup C_i$ is a diagram. Prove that $\mathrm{lk}(C_i, C_j)$ is an integer. [Harder] Fill in the details of the proof that linking numbers are isotopy invariants.

Exercise 1.5. Prove the easy direction of Reidemeister's theorem: Suppose that K, K' are knots with diagrams D, D'. If D, D' are related be a single Reidemeister move then K is isotopic to K'.

Exercise 1.6. Show that the R_∞ move can be obtained as via a sequence of the standard four moves. [Harder] Give bounds on the number of R_3 and R_2 moves needed in terms of the given diagram.

Exercise 1.7. Show that isotopy of knots is an equivalence relation.

Exercise 1.8. Show that the figure eight is isotopic to its mirror image. (Use a piece of string!) Now draw a sequence of Reidemeister moves to prove that the two knots are isotopic. (Hint: Exercise 1.6 may be useful.)

Exercise 1.9. The figure eight has two orientations. Are these isotopic? If so, provide a sequence of Reidemeister moves.

Exercise 1.10. Provide a short proof that the unlink and the Hopf link are not isotopic. Think about how you would prove that the unlink and the Whitehead link are not isotopic.

Exercise 1.11. As done in the notes for the trefoil and the figure eight, find non-trivial colorings of the Whitehead link. Careful: you cannot divide by two in the ring $\mathbb{Z}/2m\mathbb{Z}$. (That is, when the modulus is even.)