

These exercises are mainly taken from the last week's lectures. Please let me know if any of the problems are unclear or have typos.

**Exercise 10.1.** [Harder] Give an upper bound on  $g(K)$  where  $K$  is a four-plat in standard position.

**Exercise 10.2.** Show that the trefoil, the figure eight, the  $6_1$  knot and the  $9_{46}$  knot all have genus one.

**Exercise 10.3.** Looking at KnotInfo, find a relation between knot genus and the span of the Alexander polynomial.

**Exercise 10.4.** If  $\gcd(p, q) = 1$  then let  $T(p, q)$  be the knot on the standard two-torus with *slope*  $p/q \in \mathbb{Q} \cup \{\infty\}$ .

- For all  $n \in \mathbb{Z}$  the knots  $T(1, n)$  and  $T(n, 1)$  are isotopic to the unknot.
- Draw  $T(4, 3)$ , the  $(4, 3)$  torus knot.
- (Hard) Show that  $4_1$  is not a torus knot.
- Show that  $T(p, q)$  is isotopic to  $T(q, p)$ .
- Show that if  $\gcd(p, q) = n$  then the link  $T(p, q)$  has  $n$  components.

**Exercise 10.5.** [Harder] Prove that any knot  $K$  has  $\nabla_K$  being a polynomial in  $z^2$ . Here  $\nabla_K$  is the Conway polynomial of  $K$ .

**Exercise 10.6.** Compute the Conway polynomial of the trefoil, figure eight knot, and the Whitehead link.

**Exercise 10.7.** Show that the Conway polynomial does not distinguish mirror image knots. Does it distinguish mirror image links?

**Exercise 10.8.** Give formulas for  $P_{mK}$  and  $P_{rK}$  in terms of  $P_K$ , the HOMFLY polynomial of  $K$ .