These exercises are mainly taken from the third week's lectures. Please do let me know if any of the problems are unclear or have typos.

For the next several exercises, A_+ is the matrix of crossing equations and A is the matrix obtained by deleting a row and column from A_+.

Exercise 3.1. Compute A_+ for the twist knots, T_k.

Exercise 3.2. Show that $|\det(A)|$ is independent of the choice of row and column deleted from A^+.

Exercise 3.3. [Harder] Show that the Smith normal form of A is independent of the choice of row and column deleted from A^+. (Hint: The fact proved in class, that it is possible to choose the signs of the rows so that the entries of any single column sum to zero, may be helpful.)

Exercise 3.4. In our use of Cramer's rule (to find possible solutions to the coloring equations) the choice of b was not specified. Setting $b = (1, 0, \ldots, 0)$ we find that $x_k = (-1)^{k+1}\mathrm{Minor}_{1,k}(A)$ and also that $\det(A) = A^1 \cdot x$ where A^1 is the first row of A. Use this to find a coloring modulo $\det(K)$ of the knot 6_3.

Exercise 3.5. Check that if a diagram is *alternating* (every overcrossing arc goes over exactly one crossing) then the variables may be ordered so that the matrix A^+ has twos along the diagonal.

Exercise 3.6. Do Exercise 11 on Sanderson's example sheet:

`http://www.warwick.ac.uk/~maaac/examples2.html.`

Here a quadrilateral decomposition is the planar graph *dual* to the shadow.

Exercise 3.7. Compute the determinant of the 5_1 knot (the cinefoil) and the twist knots. Notice that the first two twist knots are the trefoil and the figure eight.

Exercise 3.8. Let $T(2, 4)$ be the $(2, 4)$–torus link. Let W be the Whitehead link. Show that $\det(T(2, 4)) = 4$ while $\det(W) = 8$.

Exercise 3.9. Show that $P = P(-2, 3, 5)$ has determinant equal to one.

Exercise 3.10. [Harder] Compute the coloring group of the pretzel link $P = P(p, q, r)$. Determine which triples (p, q, r) give the trivial group.

Exercise 3.11. Prove that the coloring group is an isotopy invariant. To do this show that if two diagrams D and D' differ by a single Reidemeister move then $\mathrm{Col}(D)$ and $\mathrm{Col}(D')$ are isomorphic.

Exercise 3.12. Compute the coloring group $\mathrm{Col}(L)$, where L is the 12–crossing two-component "boundary link" with determinant zero shown in class.

Exercise 3.13. Check that 8_2 and 8_{17} have isomorphic coloring groups.