These exercises are mainly taken from the forth week's lectures. Please do let me know if any of the problems are unclear or have typos.

Exercise 4.1. If you haven't already done so, check that the figure eight knot (4_1) is achiral.

Exercise 4.2. Suppose that $P = P(p_1, p_2, \ldots, p_n)$ is a pretzel knot. Show that if one of the p_i is even then P is invertible. [Hard] Are all pretzel knots invertible?

Exercise 4.3 (Harder)**.** In class we proved that if D is a diagram of an oriented knot and if one travels along the knot labelling the crossings (so that every crossing receives two numbers) then every crossing receives exactly one odd and one even label. Review the proof of this fact, or find your own proof. [Note that this proof is an important step in the construction of Dowker codes.]

Exercise 4.4. The knots 6_1, 6_2, 6_3, have codes

$$[4, 8, 12, 10, 2, 6] \quad [4, 8, 10, 12, 2, 6] \quad [4, 8, 10, 2, 12, 6]$$

respectively. Draw these and check that they are isotopic to the standard diagrams.

Exercise 4.5. Recognize the knot with code $[6, 8, 10, 2, 4]$.

Exercise 4.6. Compute the codes for the granny and reef knots.

Exercise 4.7. Prove that there are at most $2^n \cdot n!$ knots, up to isotopy, with n or fewer crossings. [Hard] Can you also give an exponential lower bound?

Exercise 4.8. Determine the units of the ring $\mathbb{Z}[t, t^{-1}]$.

Exercise 4.9. Compute the Alexander polynomial for the trefoil knot by first computing the matrix of crossing equations. You should find that $\Delta_T(t) = t - 1 + t^{-1}$, up to multiplication by units in the ring $\mathbb{Z}[t, t^{-1}]$.

Exercise 4.10. Show that $w(R)$, the winding number of the oriented shadow around the region R, is well defined. (Hint: this is very similar to the proof, given in class, that the parity $e(R)$ is well-defined.) Show that if R, R' are adjacent then $w(R) = w(R') \pm 1$. Show that $w(R) = e(R) \mod 2$ where $e(R)$ is the parity of the region R.