These exercises are mainly taken from the seventh week's lectures. Please let me know if any of the problems are unclear or have typos.

Exercise 7.1. Show that the number of Seifert circles of a diagram is not a knot invariant.

Exercise 7.2. Show that for any knot diagram the number of Seifert circles and the writhe have opposite parity.

Exercise 7.3. Provide the details the lemma claimed in class: If D is a diagram, and D' is the diagram after a conflict has been resolved, then D' has exactly one less disagreement than D.

Exercise 7.4. Let D be the standard diagram of the $(2,4)$–torus link where one component has clockwise and the other has anticlockwise orientation. Give a sequence of R_0, R_2, and R_∞ moves turning D into a braid closure. Write down the resulting braid word. Compare your result to those of others in the class. Essay an explanation of the many different (and correct) answers possible.

Exercise 7.5. For each knot K, up to six crossings, find a braid word σ_K so that K is isotopic to the braid closure of σ_K. Do this by orienting the knot and using the algorithm given in class. (If you find another method let me know!) Compare the words you find with those found by other people in the class.

Exercise 7.6. Show that $\mathrm{br}(K\#L) \leq \mathrm{br}(K) + \mathrm{br}(L) - 1$; that is, braid index is subadditive. [Hard] Can this inequality be reversed?

Exercise 7.7. [Harder] Show that in addition to the R_2 and R_3 moves the Kauffman bracket is also invariant under the R_∞ move. Reviewing your answer to Exercise 1.6 may be helpful. (In fact, this justifies adding the R_∞ move to the definition of regular isotopy.)

Exercise 7.8. Compute the Kauffman bracket of the $(2,p)$–torus links. [Harder] Do the same for twist knots.

Exercise 7.9. Suppose that D, E are diagrams, $D \amalg E$ is their disjoint union, and $D\#E$ is their connect sum. Use the state sum formulation of the Kauffman bracket to show that

- $\langle D \amalg E\rangle = \langle D\rangle\langle E\rangle \cdot (-A^2 - A^{-2})$,
- $\langle D\#E\rangle = \langle D\rangle\langle E\rangle$.

Exercise 7.10. Prove that the writhe of a diagram is a regular isotopy invariant.