

These exercises are mainly taken from the eighth week's lectures. Please let me know if any of the problems are unclear or have typos.

**Exercise 8.1.** Verify Dehn's result: the right and left trefoils are not isotopic.

**Exercise 8.2.** Prove that the Kauffman polynomial  $X_K$  of an amphichiral knot  $K$  is symmetric. Check this for the figure eight knot.

**Exercise 8.3.** A question asked by a student in class: Is there a chiral knot  $K$  so that  $X_K$  is symmetric? (The website KnotInfo suggests looking at the  $9_{42}$  knot.)

**Exercise 8.4.** Compute the Jones polynomial for the figure eight knot by first computing the Kauffman bracket, computing the writhe, and making the correct substitution. (The website KnotInfo gives  $V_K = t^2 - t + 1 - t^{-1} + t^{-2}$ .)

**Exercise 8.5.** [Hardest] Show that  $V_K = 1$  if and only if  $K$  is the unknot.

**Exercise 8.6.** Suppose that  $K, L$  are oriented links. Show that

- $V_{K \cup L} = -(t^{1/2} + t^{-1/2})V_K V_L$  where  $K \cup L$  is the disjoint union of  $K$  and  $L$ .
- $V_{K \# L} = V_K V_L$  where  $K \# L$  is the connect sum of  $K$  and  $L$ .

**Exercise 8.7.** Compute the Jones polynomial of the figure eight knot via the skein relation.

**Exercise 8.8.** Compute the Jones polynomials of the  $(2, p)$ -torus knots. [Harder] Do the same for the twist knots.

**Exercise 8.9.** Suppose that  $p, q$  are non-zero Laurent polynomials in the variable  $t$ . Check that, reminiscent of the logarithm,  $\text{span}(pq) = \text{span}(p) + \text{span}(q)$ . Thus, we take  $\text{span}(0) = -\infty$  as a convention.

**Exercise 8.10.** Compute the highest and lowest powers of  $V_K$  for all prime knots up to six crossings. Check that the span agrees with the crossing number. (PlanetMath.org asserts that  $8_{19}$  is the first non-alternating knot in Rolfsen's table.)

**Exercise 8.11.** Compute the Jones polynomial of the following link:

