MA3F2 Exercise sheet 8.

These exercises are mainly taken from the eighth week's lectures. Please let me know if any of the problems are unclear or have typos.

Exercise 8.1. Verify Dehn's result: the right and left trefoils are not isotopic.

Exercise 8.2. Prove that the Kauffman polynomial X_K of an amphichiral knot K is symmetric. Check this for the figure eight knot.

Exercise 8.3. A question asked by a student in class: Is there a chiral knot K so that X_K is symmetric? (The website KnotInfo suggests looking at the 9_{42} knot.)

Exercise 8.4. Compute the Jones polynomial for the figure eight knot by first computing the Kauffman bracket, computing the writhe, and making the correct substitution. (The website KnotInfo gives $V_K = t^2 - t + 1 - t^{-1} + t^{-2}$.)

Exercise 8.5. [Hardest] Show that $V_K = 1$ if and only if K is the unknot.

Exercise 8.6. Suppose that K, L are oriented links. Show that

- $V_{K \cup L} = -(t^{1/2} + t^{-1/2})V_K V_L$ where $K \cup L$ is the disjoint union of K and L.
- $V_{K\#L} = V_K V_L$ where K#L is the connect sum of K and L.

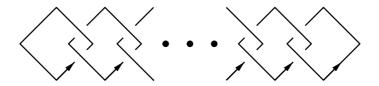
Exercise 8.7. Compute the Jones polynomial of the figure eight knot via the skein relation.

Exercise 8.8. Compute the Jones polynomials of the (2, p)-torus knots. [Harder] Do the same for the twist knots.

Exercise 8.9. Suppose that p, q are non-zero Laurent polynomials in the variable t. Check that, reminiscent of the logarithm, $\operatorname{span}(pq) = \operatorname{span}(p) + \operatorname{span}(q)$. Thus, we take $\operatorname{span}(0) = -\infty$ as a convention.

Exercise 8.10. Compute the highest and lowest powers of V_K for all prime knots up to six crossings. Check that the span agrees with the crossing number. (PlanetMath.org asserts that 8_{19} is the first non-alternating knot in Rolfsen's table.)

Exercise 8.11. Compute the Jones polynomial of the following link:



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