Please let me know if any of the problems are unclear or have typos.

Exercise 1.1. Suppose that M^n is a manifold. Prove directly from the definition that ∂M is either empty or is an $n-1$–manifold. Prove that $\partial\partial M$ is empty.

Exercise 1.2. Show that the definitions of S^n given in class (as a submanifold of $\mathbb{R}^{n+1}$, as the one-point compactification of $\mathbb{R}^n$, and as the double of an n–ball) are equivalent.

Exercise 1.3. Give a map $T^n \to \mathbb{R}^{n+1}$ that is an *embedding*: a diffeomorphism onto its image. Show that any compact n–manifold embedded in $\mathbb{R}^n$ has non-empty boundary; deduce that T^n does not embed in in $\mathbb{R}^n$.

Exercise 1.4. [Hard] Verify the classification, up to homeomorphism, of compact connected 1–manifolds. For a detailed outline of the argument, see David Gale's article "The classification of 1-manifolds: a take-home exam", in the American Mathematical Monthly.

Exercise 1.5. Show that, up to homeomorphism, connect sum is commutative, associative, and that the n–sphere is an identity element: $M^n \# S^n \cong M^n$.