

Please let me know if any of the problems are unclear or have typos.

Exercise 10.1. Classify the Seifert fiberings of $T \times I$, up to isotopy. (The classification up to homeomorphism is much simpler.)

Exercise 10.2. [Hard] Show that the Pachner moves (also called *bisteller flips*) do not change the quantity $\sum_{k=0}^n (-1)^k |T^{(k)}|$.

Exercise 10.3. [Part of the proof of Proposition 1.13] Suppose that (M, \mathcal{F}) is a Seifert fibered space and $T \subset \partial M$ is a torus. Suppose that $(D, \partial D) \subset (M, T)$ is a compressing disk. Deduce that the orbifold $B = M/S^1$ is a disk with at most one cone point. Deduce $M \cong D \times S^1$.

Exercise 10.4. Suppose that T_i , for $i = 0, 1$, are copies of $M^2 \times I$. Let $A_i = \partial_h T_i$. Show that for any homeomorphism $\phi: A_0 \rightarrow A_1$ the manifold $T_0 \cup_\phi T_1$ is homeomorphic to $K^2 \times I$. Classify Seifert fiberings on $K \times I$, up to isotopy.

Exercise 10.5. [Reading exercise] Read the proof of the uniqueness statement in Theorem 1.9 in Hatcher's notes.

Exercise 10.6. Let D be the dodecahedron. Let $P = D/\sim$ be the space obtained by gluing opposite faces via $1/10^{\text{th}}$ right-handed rotation. Show that the result is a three-manifold. Give a presentation of $\pi_1(P)$. Check that $H_1(P, \mathbb{Z})$ is trivial. [What manifold do you obtain if you instead use rotation by $1/2$? The manifold obtained via $3/10^{\text{th}}$ rotation is harder to understand.]

Exercise 10.7. Suppose that M is irreducible, connected, $T \subset \partial M$ is a torus, and $\pi_1(M) \cong \mathbb{Z}$. Prove that $M \cong D \times S^1$.

Exercise 10.8. Suppose that F is a properly embedded 2-sided surface in M^3 . Suppose that $\Gamma = \ker(\pi_1(F) \rightarrow \pi_1(M))$ is nontrivial. Then there is an essential, simple loop in Γ .

Exercise 10.9. Show that Exercise 10.8 is false if we remove the two-sided hypothesis.