Please let me know if any of the problems are unclear or have typos.

Exercise 11.1. Suppose that F is a closed connected orientable surface other than the two-sphere. Give a hierarchy for $F \times I$. Give two distinct hierarchies for $F \times S^1$.

Exercise 11.2. Suppose that C is a compression body. Show that any essential surface in C is either

- a compressing disk for $\partial_+ C$,
- a component of $\partial_- C$, or
- an annulus that meets both $\partial_\pm C$.

Exercise 11.3. Suppose that S is essential in M, a Haken three-manifold. Let $N = M - n(S)$. Let $F \subset \partial N$ be an component and let G be the result of maximally compressing F into N, always in the same direction, and then discarding two-sphere components. Show that G is incompressible in M.

Exercise 11.4. Suppose that Q is a regular n–gon in the plane. Let V be the vertices of Q. Let $B = Q \times I$ be a three-ball, with boundary pattern $P = (\partial Q cross \{0, 1\}) \cup (V \times I)$. Show that, if $n > 3$, that P is an essential boundary pattern. Next, classify pattern-essential surfaces in (B, P).

Exercise 11.5. Deduce the Disk Theorem from Theorem 9.1 in Lackenby's notes.

Exercise 11.6. Let $K \subset S^3$ be the 5_2 knot, as shown in Figure 1. Let $S \subset X = X_K$ be the shaded surface shown in the figure. Check that S is orientable. As done in class, let $M_1 = X - n(S)$. Check that M_1 is a handlebody, and carefully draw the boundary pattern P_1. Now cut along disks to get M_2. Check that M_2 is a three-ball, and carefully draw the resulting pattern P_2. Using the above or otherwise prove that that S is essential.

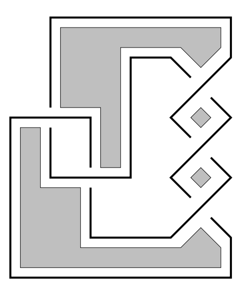

Figure 1: The 5_2 knot, with Seifert surface shaded.