Please let me know if any of the problems are unclear or have typos.

Exercise 2.1. Extend the definition of connect sum to non-orientable surfaces and prove that $\#_3\mathbb{P} \cong \mathbb{T}\#\mathbb{P}$.

Exercise 2.2. [Alexander Trick] Suppose that $\phi\colon S^2 \to S^2$ is a homeomorphism and $\mathrm{Id}\colon S^2 \to S^2$ is the identity. Find an explicit homeomorphism between $M = \mathbb{B}^3 \cup_\phi \mathbb{B}^3$ and $S^3 = \mathbb{B}^3 \cup_{\mathrm{Id}} \mathbb{B}^3$. (It follows, in dimension three, that M is diffeomorphic to the three-sphere. The Alexander trick works in all dimensions but the promotion to smoothness does not. See Milnor's paper "On manifolds homeomorphic to the 7–sphere".)

Exercise 2.3.

- Show that if M^3 is irreducible then M is prime.
- Show that if M is orientable and $S \subset M$ is a non-separating two-sphere embedded in M then $M = S^2 \times S^1 \# N$.
- Suppose that M^3 is orientable. Then: M^3 is prime and reducible iff $M \cong S^2 \times S^1$. Prove the forward direction.

Exercise 2.4. [Medium] Prove the backwards direction of part (iii) of Exercise 2.3. [Idea: rewrite the proof of Alexander's theorem.]

Exercise 2.5. [Medium] Prove the Jordan-Schoenflies theorem: every smoothly embedded S^1 in $\mathbb{R}^2$ bounds a disk. [Idea: rewrite the proof of Alexander's theorem for dimension two.]

Exercise 2.6. Suppose that $K^1 \subset S^3$ is a knot. Let $X_K = S^3 - n(K)$ be the *knot complement*. Show that ∂X_K is compressible iff K is the unknot (isotopic to a round circle).

Exercise 2.7. Suppose that M is an irreducible three-manifold and $F, G \subset \partial M$ are disjoint, incompressible subsurfaces. Suppose that $\phi\colon F \to G$ is a homeomorphism. Show that M/ϕ is irreducible.