Please let me know if any of the problems are unclear or have typos.

Exercise 4.1. Show that there are exactly two inequivalent $\mathbb{B}^k$–bundles over S^1. The same holds for S^1– and S^2–bundles over S^1.

Exercise 4.2. Prove that M^n is orientable if and only if for every simple closed curve $\alpha \subset M$ the regular neighborhood $N(\alpha)$ is a trivial bundle.

Exercise 4.3. Classify, up to bundle equivalence, two-fold covers of $\mathbb{T}^2$. Do the same for S_2, the closed orientable surface of genus two. Which of these can you give pictures for?

Exercise 4.4. Suppose that $\rho\colon T \to F^2$ is an $\mathbb{B}^1$–bundle. Show that the vertical boundary $\partial_v T$, the horizontal boundary $\partial_h T$, and the zero-section are all incompressible in T. (Here we exclude the case of the product I–bundle over D^2.) On the other hand: show that if F has boundary and T is orientable, then T is a handlebody. Thus ∂T is often compressible.

Exercise 4.5. Suppose that M is an irreducible three-manifold and $F, G \subset \partial M$ are disjoint, incompressible subsurfaces. Suppose that $\phi\colon F \to G$ is a homeomorphism. Suppose that $H \subset M$ is a properly embedded incompressible surface and disjoint from F and G. Show that the image of H is M/ϕ is incompressible. Deduce that if that $\rho\colon T \to S^1$ is an F^2–bundle then all fibers (point preimages) are incompressible.

Exercise 4.6. [Easy] Suppose that $V = V_2$ is a handlebody of genus two. Prove or find a counterexample: any surface properly embedded in V is compressible or is ambient isotopic into a regular neighborhood of ∂V.

Exercise 4.7. [Medium] Suppose that T is a finite triangulation. Give necessary and sufficient combinatorial conditions so that $||T||$ is homeomorphic to a topological manifold M^n, for $n \leq 3$.

Suppose that (F, T) is a triangulated surface. Call a simple closed curve $\alpha \subset F$ *normal* if α is transverse to the skeleta of T and $\alpha \cap \Delta$ is a finite collection of normal arcs, for every triangle $\Delta^2 \subset F$. The *weight* of α is $w(\alpha) = \left|\alpha \cap T^{(1)}\right|$.

Exercise 4.8. Suppose that (F, T) is the boundary of a three-simplex. Show that if α is a normal curve and if α meets every edge of $T^{(1)}$ at most once then α has weight three or four. Deduce that there are seven normal disks in a tetrahedron.

Exercise 4.9. In the proof of Haken-Kneser finiteness we defined N to be the closure of the union, over all $\Delta \in T^{(3)}$, of all components of $\Delta - S$ meeting F. Prove that N is an I–bundle and either N is ambient isotopic to $N(F)$ *or* F is two-sided and parallel to $\partial_h N$.