Please let me know if any of the problems are unclear or have typos.

Exercise 5.1. For each of the two triangulations shown in Figure 1 prove that the underlying space is a three-manifold. Compute the fundamental groups and identify each manifold (by giving a homeomorphism).

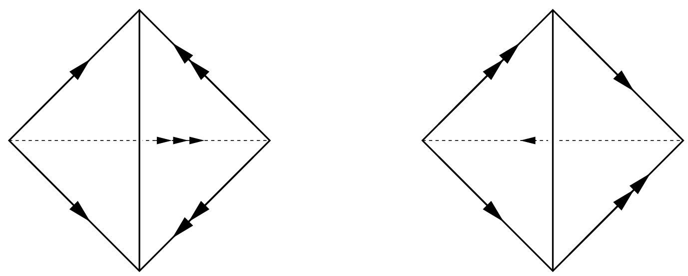

Figure 1: Each one-tetrahedron triangulation has exactly one face pairing between the back two faces.

Exercise 5.2. For each of the two triangulations shown in Figure 2 prove that the underlying space is a three-manifold. Compute the fundamental groups and identify each manifold (by giving a homeomorphism).

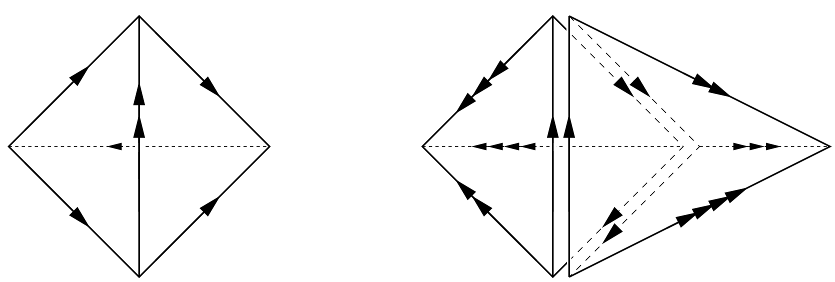

Figure 2: The left triangulation has two face pairings gluing the back two and the front two faces. The right triangulation has four face pairings.

Exercise 5.3. Classify, up to normal isotopy, all normal curves in (F^2, T) where:

- (F, T) is the usual triangulation of the torus, with two triangles. [Medium]
- (F, T) is the usual triangulation of the Klein bottle with two triangles. [Medium-hard]
- (F, T) is the two-sphere, triangulated as the two-skeleton of a tetrahedron. [Hard]

Exercise 5.4. For the cubing shown in Figure 3 prove that the underlying space Q is a three-manifold. Compute the fundamental group $\Gamma = \pi_1(Q)$ and show that Γ is finite and not Abelian. [Harder: Compute the universal cover $\widetilde{Q} \to Q$ and the associated action of Γ on $\widetilde{Q}$.]

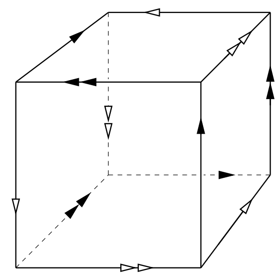

Figure 3: The *quarter-turn* space: opposite faces are idenified by a right handed quarter-turn. What do you get if you use a one-half turn instead? What manifolds arise from similarly nice face pairings of other Platonic solids?

Exercise 5.5. Suppose that $F \subset (M, T)$ is a closed incompressible embedded surface. Suppose that M is irreducible. Show that F is isotopic to a normal surface. (That is, there is a map $H\colon F \times I \to M$ so that $H_0 = \mathrm{Id}\,|F$, H_t is an embedding for all t, and $H_1(F)$ is normal.) Can you extend your proof to the case where F has boundary and is properly embedded?

Exercise 5.6. [Easy] Let $B_n = \#_n \mathbb{B}^3$ be the n–times *punctured three-sphere*. Here are two statements left over from the proof of existence of prime factorizations.

- Suppose that P, Q are three-manifolds and $\phi\colon S \to T$ is a homeomorphism of two-sphere boundary components $S \subset P$, $T \subset Q$. Prove that P, Q are both punctured three-spheres if and only if $P \cup_\phi Q$ is a punctured three-sphere.
- If M has n boundary components that are two-spheres then $M \cong N \# B_n$ where N has no two-sphere boundary components.