

Please let me know if any of the problems are unclear or have typos.

Exercise 6.1. [Easy] Suppose that $\alpha \subset \partial\Delta$ is a simple closed curve in the boundary of a tetrahedron Δ . Suppose that α is transverse to $\Delta^{(1)}$ and meets some edge $e \subset \Delta^{(1)}$ in at least two points. Show that there is a component $d \subset e - n(\alpha)$ disjoint from $\Delta^{(0)}$.

Exercise 6.2. [Old] Suppose that $F \subset (M, T)$ is a closed incompressible embedded surface. Suppose that M is irreducible. Show that F is isotopic to a normal surface. [Note that this is a duplicate of Exercise 5.6 from last week.]

Exercise 6.3. [Easy] Using the definition via triangulations or otherwise, show that lens spaces are orientable.

Exercise 6.4. [Easy] Show that $P^3 \cong L(2, 1)$. Thus P^3 is orientable.

Exercise 6.5. [Medium] Show that there are exactly two I -bundles over P^2 , up to equivalence. Show that the non-trivial bundle is homeomorphic to $P^3 - \text{interior}(B^3)$ (a once-punctured projective space).

Exercise 6.6. We gave three definitions of a lens space in class: as the quotient of the three-sphere, as the gluing of solid tori, and as the quotient of a lens (that is, of a three-ball). Show that the three definitions are equivalent by providing the necessary homeomorphisms. Mind your p 's and q 's!

Exercise 6.7. Suppose that $F \subset M$ is a properly embedded surface. Suppose that the induced map on fundamental groups $\iota_*: \pi_1(F) \rightarrow \pi_1(M)$ is injective. Show that F is incompressible.

Exercise 6.8. [Medium] Show that the three-sphere S^3 and the three-ball B^3 are atoroidal. Show that the solid torus $D \times S^1$ is atoroidal. Show that T^3 is toroidal.