Please let me know if any of the problems are unclear or have typos.

Exercise 7.1. Suppose that M, N are three-manifolds, $F, G \subset \partial M, \partial N$ are components of their boundaries, and $\phi, \phi' \colon F \to G$ are isotopic homeomorphisms. Define $Y = M \cup_\phi N$ and $Y' = M \cup_{\phi'} N$. Show that Y is homeomorphic to Y'.

Exercise 7.2. Using Exercise 5.3 or otherwise, classify essential curves in the Klein bottle K^2.

Exercise 7.3. Let $V = D \times S^1$ be a solid torus and let T be the orientation I–bundle over K^2. Suppose that $\phi \colon \partial V \to \partial T$ is a homeomorphism. Then $M_\phi = V \cup_\phi T$ is called a *prism manifold*. Show that prism manifolds are double covered by lens spaces (or $S^2 \times S^1$).

Exercise 7.4. [Hard] Show that $S^2 \times S^1$ and also the quarter-turn space are prism manifolds.

Exercise 7.5. Suppose that $(M, \mathcal{F})$ is a oriented three-manifold equipped with a Seifert fibering. Show that the singular fibers are isolated and lie in the interior of M. Thus, if M is compact then there are only finitely many singular fibers.

Exercise 7.6. Find the Seifert fibering of the knot complement X_K where $K = K(p, q)$ is a torus knot. Compute the number of singular fibers, their multiplicities, and the base orbifold.

Exercise 7.7. Find a Seifert fibering of the lens space $L(p, q)$. Compute the number of singular fibers and multiplicities as well as the base orbifold of your fibering.

Exercise 7.8. Consider the two tetrahedra shown Figure 1. Does the given triangulation determine a three-manifold? If it does not, give a reason. If it does, recognize the manifold.

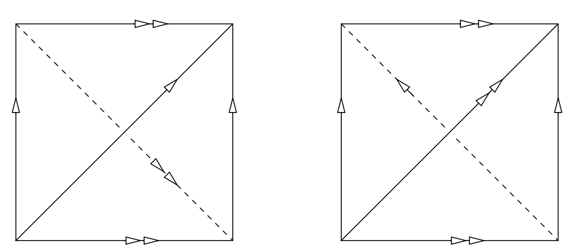

Figure 1: The front faces on the left are glued to the back faces on the right; likewise the back faces on the left are glued to the front faces on right. In this example, the face pairings are determined by the edge identifications but this does not hold in general.