

Please let me know if any of the problems are unclear or have typos.

Exercise 8.1. Let $P \times I$ be the orientation I -bundle over the projective plane. Show that $P \times I$ is homeomorphic to $P^3 \# B^3$, a punctured projective space.

Exercise 8.2. Suppose that $S \subset V$ is a surface properly embedded in a handlebody V . Show that S is compressible, is boundary compressible, is a disk or is a sphere.

Exercise 8.3. [Easy] Suppose that $S \subset M$ is a connected incompressible surface (not a disk). Show that if $\partial S \neq \emptyset$ then every component of ∂S is essential in ∂M .

Exercise 8.4. Suppose that $F \subset M$ is incompressible. Suppose that the bigon D is a boundary compression for F . Show that F_D , the surgery of F along D , is again incompressible.

Exercise 8.5. Suppose that $\rho: T \rightarrow F$ is an I -bundle. Show that $\partial_h T$ and $\partial_v T$ are boundary incompressible. [If $\partial F \neq \emptyset$ and if $\partial_h T$ is not connected then the components are, individually, boundary compressible. Likewise, the “1/2-section” (zero-section) is boundary compressible in T .]

Exercise 8.6. [Hard] Suppose that $\rho: T \rightarrow F$ is an I -bundle. Show that any essential surface $S \subset T$, that is not a disk, may be isotoped to be either vertical or horizontal. [Hint: there is a proof modelled on the proof of Proposition 1.11 from Hatcher.]