Please let me know if any of the problems are unclear or have typos.

Exercise 9.1. Suppose that $\rho\colon X \to F$ is an I–bundle. Show that X is atoroidal if it is not homeomorphic to $T^2 \tilde{\times} I$.

Exercise 9.2. List all compact, connected 2–orbifolds B with $\chi_{\text{orb}}(B) = 0$. Challenge: can you do the same when the orbifold Euler characteristic is positive?

Exercise 9.3. Find all orbifold double covers of $D^2(2, 2)$ and of $S^2(2, 2, 2, 2)$.

Exercise 9.4. Suppose that S is a horizontal surface in a Seifert fibered space M. Let α be any generic fiber and set $d = |S \cap \alpha|$. Let $B = M/S^1$. Prove that $\chi(S) = d \cdot \chi_{\text{orb}}(B)$.

Exercise 9.5. Show that lens spaces are atoroidal.

Exercise 9.6. Let $P = \#_3 D^2$ be a *pair of pants*. Classify, up to proper isotopy, all essential loops and arcs in P.

Exercise 9.7. Define the *solid Klein bottle* to be $V = D \tilde{\times} S^1 = D \times I/(z, 1) \sim (\bar{z}, 0)$; that is, we glue by a reflection. Show that V admits a partition into circles yet is not a Seifert fibered space, according to Hatcher's definition.

Exercise 9.8. Suppose that $K = K_{p,q}$ is a (p, q)–torus knot, with $|p|, |q| > 1$. Let $X = X_K$ be the knot exterior. Using the Seifert fibering prove the following statements.

- Any horizontal surface has negative Euler characteristic.
- There are only two essential vertical 2–sided surfaces in X: the boundary parallel torus and a separating annulus A so that $X - n(A)$ is a pair of solid tori.

Deduce that ∂X is incompressible, X is atoroidal, and $A \subset X$ is the unique essential annulus as stated in Lecture 17. It follows that the numbers $|p|, |q|$ are invariants of the homeomorphism type of X.

Exercise 9.9. Take K, X as in Exercise 9.8. Show that there is a horizontal surface $F \subset X$ with the following properties.

- The surface F *spans* K. That is, the boundary ∂F is a single curve, meeting the meridian of $N(K)$ is a single point.
- The surface F has genus $g(F) = (p - 1)(q - 1)/2$.
- There is no orientable spanning surface for K with lower genus.

Since F is non-separating and horizontal it follows that X is an F–bundle over the circle, with periodic monodromy.