Please let me know if any of the problems are unclear or have typos.

Exercise 10.1. Suppose $F = S_{g,n,c}$ is a compact connected surface with genus g, with $n > 0$ boundary components, and with c cross-caps. Show F can be obtained as a *disk-with-bands*: a disk D with *bands* $H_i \cong I^2$ attached to D along $I \times \partial I$. How do the number and kind of bands determine g, n, and c?

Exercise 10.2. Suppose $F = S_{g,n,c}$ is a compact connected surface, as in Exercise 10.1, with $n > 0$. Show that $\pi_1(F, x)$ is a free group. The rank of the free group is equal to the number of bands needed to build F as a disk-with-bands.

Exercise 10.3. Suppose $p\colon T \to F$ is an S^1–bundle over F, a compact connected surface. For $U \subset F$ we define $T|U = p^{-1}(U)$. Let x be a point of F and let $\{\beta_i\}$ be a collection of simple loops based at x that generate $\pi_1(F, x)$. Define $\tau_p\colon \pi_1(F, x) \to \{\pm 1\}$ by

$$\tau_p(\beta_i) = \begin{cases} +1 & \text{if } T|\beta_i \text{ is a torus,} \\ -1 & \text{if } T|\beta_i \text{ is a Klein bottle} \end{cases}$$

and extending multiplicatively to elements of $\pi_1(F, x)$.

- Show τ_p is well-defined.
- Show, when F has boundary, that τ_p determines $p\colon T \to F$, up to isomorphism.
- Give examples to show τ_p does not determine $p\colon T \to F$, even up to isomorphism, when $\partial F = \emptyset$.

Exercise 10.4. Suppose F is a compact connected surface. Define $\sigma\colon \pi_1(F, x) \to \{\pm 1\}$ by $\sigma(\gamma) = 1$ if and only if the path γ preserves orientation. Suppose $p\colon T \to F$ is an S^1–bundle. Show T is orientable if and only if $\tau_p = \sigma$.

Exercise 10.5. Suppose $p\colon T \to F$ is an S^1–bundle. Show, when $\partial F \neq \emptyset$, that p admits a section s. Furthermore s is unique, up to isomorphism of p.