Please let me know if any of the problems are unclear or have typos.

Exercise 11.1. [Medium] After recalling the definitions show that the lens space $L(2,1)$, the projective space $\mathbb{RP}^3$, the Lie group $\mathrm{SO}(3)$, and the unit tangent bundle $\mathrm{UT}\, S^2$ are all homeomorpic. The Hopf fibration on S^3 descends to a Seifert fibered structure on $L(2,1)$; this is isomorphic to the natural S^1–fibration of $\mathrm{UT}\, S^2$. The Euler number is two.

Exercise 11.2. We say an unoriented simple closed curve $\alpha \subset \mathbb{T}^2$ is *essential* if α does not bound a disk in $\mathbb{T}^2$. Isotopy of essentially curves is an equivalence relation; the classes of the relation are called *slopes*.

Show the set of slopes is naturally in bijection with $\mathbb{Q} \cup \{1/0\}$. Show if α and β correspond to p/q and r/s then the algebraic intersection number satisfies

$$|\alpha \cdot \beta| = |ps - qr|,$$

after chosing orientations for α and β.

Exercise 11.3. Suppose that M is a Seifert fibered space. Let $B = M/S^1$. There are three orientations to consider: orientability of M as a three-manifold, orientability of B as a two-orbifold, and orientability of the fibration. (The last is equivalent to the fibration arising as the orbits of an S^1-action on M.) Show, via examples, that none, all, or any two of these can be twisted. However, it is impossible for exactly one of them to be twisted.

Exercise 11.4. Check, directly from the definition, that $e(S^1 \times F) = 0$: the Euler number of a product fibration is zero.

Exercise 11.5. [Medium]

- Let M_h be the half-turn manifold from Exercise 7.6. Compute the Euler number $e(M_h)$ directly from the definition. Check also that M_h is double covered by the three-torus.
- Do the same for M_r, the quarter-turn manifold from Exercise 8.6. Show M_r is double covered by M_h.

Exercise 11.6. The Hopf fibration on S^3 descends to give a fibration of $L = L(p,q)$. Show that $e(L) = p^{\pm 1}$; determine how q affects the answer.

Exercise 11.7. [Medium] Suppose F is a connected closed surface. Equipping $\mathrm{UT}\, F$ with the usual S^1–bundle structure, show $e(\mathrm{UT}\, F) = \chi(F)$. (This fixes the sign convention for the Euler number.)