Please let me know if any of the problems are unclear or have typos.

Exercise 2.1. Prove the surface spanning the trefoil, with two disks and three half-twisted bands (as drawn in class), is homeomorphic to a *handle*: the two-torus minus an open disk.

Exercise 2.2. The classification of surfaces states that two compact connected surfaces F and G are homeomorphic if and only if F and G have the same orientability, Euler characteristic, and number of boundary components. Show by means of examples that no two of those invariants suffice.

Exercise 2.3. Suppose B is a manifold and C, D are disjoint components of ∂B. Suppose $\phi\colon C \to D$ is a diffeomorphism and define

$$B/\phi = B/\sim \quad \text{where } x \sim y \text{ if } y = \phi(x).$$

Show $\chi(B/\phi) = \chi(B) - \chi(C)$. [Note $\chi(C) = \chi(D)$.]

Exercise 2.4. Suppose A is a manifold with boundary and $A_i = A \times \{i\}$ for $i = 1, 2$. Define the *double* of A to be

$$D(A) = A_1 \sqcup A_2/\sim \quad \text{where } (x, 1) \sim (x, 2) \text{ if } x \in \partial A.$$

Determine which closed (compact, connected, without boundary) surfaces are doubles.

Exercise 2.5. [Medium] Prove directly from the definition that $\mathbb{M}^2$ is not orientable.

Exercise 2.6. [Medium] Prove that a surface S is non-orientable if and only if S contains an embedded Möbius band.

Exercise 2.7. Recall Singer's Theorem: If M is a simply connected manifold with a complete, locally homogeneous metric then M is homogeneous. Show, by means of an example, this fails without completeness.