Please let me know if any of the problems are unclear or have typos.

Exercise 8.1. Suppose F is a connected compact surface. A *fibration* of F is a partition of F into circles, called *fibers*, so that every fiber has a neighborhood that is a standard fibered annulus or Möbius band.

- Draw the standard fiberings of the annulus and the Möbius band.
- Show if F admits a fibration then $\chi(F) = 0$.
- Show, using the above or otherwise, if F admits a fibration then F is a torus, Klein bottle, annulus, or Möbius band.

Exercise 8.2. Suppose F and G are compact connected surfaces. Fibrations of F and G are *isomorphic* if there is a homeomorphism $h\colon F \to G$ sending fibers to fibers.

- Classify the fibrations on surfaces (as in Exercise 8.1) up to isomorphism.
- Identify which of these fibrations arise as the orbits of an S^1–action.

Exercise 8.3. Suppose F is a surface, equipped with a fibration. Let $B = F/S^1$ be the quotient space where we identify $x, y \in F$ if and only if there is a fiber containing both. Show B is a 1–orbifold. For each isomorphism class from Exercise 8.2 identify B and briefly describe the quotient map $p\colon F \to B$.

Exercise 8.4. Suppose M is a Seifert fibered space. Let $B = M/S^1$ be the quotient space where we identify $x, y \in M$ if and only if there is a fiber containing both.

- Show B receives the structure of a 2–orbifold.
- Let $p\colon M \to B$ be the quotient map. If $C \subset M$ is a critical fiber, show $p(C)$ is either a cone point or a point of the mirror boundary. Deduce B has no corner reflectors.
- A subset $V \subset M$ is *vertical* if V is a union of fibers. Show any two regular fibers of M are connected by an annulus $A \subset M$ that is vertical and disjoint from the critical fibers.

Exercise 8.5. Suppose that M is a compact Seifert fibered space and $B = M/S^1$.

- Show B is closed (as an orbifold) if and only if M is closed (as a manifold). Give an example to show that M may be closed while B has mirror boundary.
- Show that if B has mirror boundary then M is not orientable. Does the converse hold?

Exercise 8.6. Let $r\colon \mathbb{T}^2 \to \mathbb{T}^2$ be defined by $r(x, y) = (1 - y, x)$. Define

$$M_r = I \times \mathbb{T}^2 \big/ (1, p) \sim (0, r(p)).$$

Show M_r is a three-manifold. Show unions of intervals of the form $I \times \{p\}$ glue to give a Seifert fiber structure. List the critical fibers and their invariants. Compute $B = M/S^1$.