Please let me know if any of the problems are unclear or have typos.

Exercise 9.1. [Not to be turned in.] Prove the following.

- The annulus $\mathbb{A}$ and the Möbius band $\mathbb{M}$ are the only I–bundles over S^1, up to isomorphism.
- The torus $\mathbb{T}$ and the Klein bottle $\mathbb{K}$ are the only S^1–bundles over S^1, up to isomorphism.

Exercise 9.2. Suppose F is equipped with the discrete topology. Show any F–bundle map $p\colon T \to B$ is a covering map.

Exercise 9.3. Show the manifolds $S^1 \times S^2$, S^3, and $\mathbb{P}^3$ are all S^1–bundles over S^2. Show they are non-isomorphic.

Exercise 9.4. Show the Hopf fibration on S^3 descends to give a Seifert fibered space structure for $L(p,q)$. Prove the quotient orbifold $L(p,q)/S^1$ is isomorphic to S^2 or $S^2(p,p)$ as $q = 1$ or $q \neq 1$. For the latter, compute the orbit invariants of the critical fibers.

Exercise 9.5. Show, for every $1 \leq q \leq p$ with $\gcd(p,q) = 1$, there is a Seifert fibered structure on S^3 so that $S^3/S^1 = S^2(p,q)$. Compute the orbit invariants of the two critical fibers. [Challenge: Classify Seifert fibered structures on S^3.]

Exercise 9.6. Classify properly embedded arcs in $\mathbb{D}^2$, $\mathbb{A}^2$, and $\mathbb{M}^2$, up to proper isotopy.

Exercise 9.7. Suppose $k \in \mathbb{Z}$. Recall from class that $\sigma_k\colon S^1 \to \mathbb{T}^2$, given by $\sigma_k(e^{i\theta}) = (e^{ki\theta}, e^{i\theta})$, is a section of the product bundle $p\colon \mathbb{T}^2 \to S^1$. (Here $\mathbb{T} = S^1 \times S^1$ and p is projection to the second factor.) Show the following.

- For any section σ of p, there is a k so that σ is isotopic to σ_k.
- For any k, ℓ there is an isomorphism $F\colon \mathbb{T} \to \mathbb{T}$ so that $F \circ \sigma_k = \sigma_\ell$. Give F in coordinates.

Exercise 9.8. Carry out Exercise 9.7 for the bundle $p\colon \mathbb{K}^2 \to S^1$.