Please let me know if any of the problems are unclear or have typos.

Exercise 1.1. Show that the map $f\colon [0,1) \to S^1$, given by $f(t) = \exp(i\pi t) = \cos(\pi t) + i\sin(\pi t)$, is a continuous bijection, but is not a homeomorphism.

Exercise 1.2. Arrange the capital letters of the Roman alphabet, ABCDEFGHIJKLMNOPQRSTUVWXYZ thought of as graphs, into homeomorphism classes. Briefly explain your reasoning, including your choice of font.

Exercise 1.3. Recall that B^n is the closed unit ball in $\mathbb{R}^n$ while S^n is the unit sphere in $\mathbb{R}^{n+1}$. Show that no two of the interval B^1, the circle S^1, the disk B^2, and two-sphere S^2 are homeomorphic.

Exercise 1.4. As in Exercise 1.2 classify the letters of the alphabet into homeomorphism types; this time, we think of the letters as small two-dimensional neighborhoods of the given planar graphs.

Exercise 1.5. [Reading exercise. Do not turn in.] The real projective space $\mathbb{RP}^n$ is the space of lines through the origin in $\mathbb{R}^{n+1}$. This can also be described as the quotient of S^n by the antipodal map. The group $\mathrm{SO}(n)$ is the group of orthogonal n-by-n matrices with determinant one.

- Show that $\mathrm{SO}(2)$ is homeomorphic to S^1.
- Show that $\mathrm{SO}(3)$ is homeomorphic to $\mathbb{RP}^3$. It follows that $\pi_1(\mathrm{SO}(3)) \cong \mathbb{Z}/2\mathbb{Z}$. Decribe the generator of $\pi_1(\mathrm{SO}(3))$ directly, and explain why traversing this loop twice gives a homotopically trivial loop.
- [Hard.] Recall that S^3 is a group via its identification with the unit quaternions $U\mathbb{H}$. Describe, in terms of the natural coordinates, the group homomorphism $U\mathbb{H} \to \mathrm{SO}(3)$ that corresponds to the quotient map $S^3 \to \mathbb{RP}^3$.