Please let me know if any of the problems are unclear or have typos.

Exercise 10.1. [Hatcher page 157, problem 27.] Suppose that (X, A) is a pair. Prove the short exact sequence $0 \to C_n(A) \to C_n(X) \to C_n(X, A) \to 0$ splits. Does this imply $H_n(X) \cong H_n(A) \oplus H_n(X, A)$? Explain.

Exercise 10.2. Recall that $S_g = \#^g T^2$ is the closed (compact without boundary), connected, orientable surface of genus g. Prove that S_g is homeomorphic to the CW–complex discussed in class – namely the complex with a single vertex, edges $\{a_i, b_i\}$, and a single 2–cell attached via the path $a_1 b_1 \bar{a}_1 \bar{b}_1 a_2 b_2 \bar{a}_2 \bar{b}_2 \ldots a_g b_g \bar{a}_g \bar{b}_g$.

Exercise 10.3. [Medium.] Prove S^∞ is contractible.

Exercise 10.4. Compute the homology groups of P^∞, of $T^n = \times^n S^1$, and of $S^\ell \times S^m$.

Exercise 10.5. [Hatcher page 157, problems 20–23.] Suppose that X and Y are finite CW–complexes. Prove any one of the following.

- $\chi(X \times Y) = \chi(X) \cdot \chi(Y)$.
- If X is the union of subcomplexes A and B then $\chi(X) = \chi(A) + \chi(B) - \chi(A \cap B)$.
- If $p\colon Y \to X$ is an n–fold covering map then $\chi(Y) = n \cdot \chi(X)$.
- If $p\colon S_h \to S_g$ is an n–fold covering map (of surfaces) then $h = n(g-1)+1$. Show this is the only restriction.