Please let me know if any of the problems are unclear or have typos.

**Exercise 11.1.** [Hatcher page 158, problem 36. Medium.] Show $H_n(X \times S^1) \cong H_n(X) \oplus H_{n-1}(X)$, where $H_i \cong 0$ for $i < 0$ by definition. Use this to give another proof that $\chi(X \times S^1) = 0$, when $X$ is a CW–complex.

**Exercise 11.2.** [Hatcher page 157, problem 24.] Let $K_5$ and $K_{3,3}$ be, respectively, the complete graph on five vertices and the complete bipartite graph on six vertices. Show that neither can be the one-skeleton of a CW–complex structure on the two-sphere.

**Exercise 11.3.** [Medium.] Suppose that $K$ is a *knot* in $S^3$: that is, an embedding of $S^1$ into $S^3$. Let $N$ be a regular neighborhood of $K$; so $N$ is an embedding of $S^1 \times B^2$ into $S^3$. Let $X = S^3 - \text{interior}(N)$ be the closure of the complement of $N$ in $S^3$. Compute $H_*(X)$. (You may assume without proof that $X$ deformation retracts to a two-dimensional CW–complex.)

**Exercise 11.4.** Recall that $SX$ is the suspension of $X$. Use the Mayer-Vietoris sequence to give another proof that $\widetilde{H}_k(X) \cong \widetilde{H}_{k+1}(SX)$.

**Exercise 11.5.** [Hatcher page 158, problem 29. Medium.] Suppose that $S_g$ is a standardly embedded surface in $\mathbb{R}^3$. Note that $S_g$ bounds a *handlebody* $V_g$ in $\mathbb{R}^3$, and $V_g$ deformation retracts to a graph. Let $M_g$ be the space obtained by gluing two copies of $V_g$ along their boundary, by the identity map. Compute $H_*(M_g)$.