Please let me know if any of the problems are unclear or have typos.

**Exercise 3.1.** Compute the simplicial homology groups of the two-sphere, directly from the definitions, using the $\Delta$–complex structure coming from the boundary of a tetrahedron. Provide clearly labelled figures.

**Exercise 3.2.** Suppose that $X$ is a path-connected space, equipped with a $\Delta$–complex structure. Show, directly from the definitions, that $H_0^\Delta(X) \cong \mathbb{Z}$. (You may assume without proof that the one-skeleton is connected.)

**Exercise 3.3.** Suppose that $X$ is a finite, path-connected, one-dimensional $\Delta$–complex: that is, a finite connected graph. Suppose that $X$ has $E$ edges and $V$ vertices. Compute the simplicial homology groups of $X$.

**Exercise 3.4.** [Do not turn in.] Compute the reduced singular homology groups of a point, directly from the definitions.

**Exercise 3.5.** [Challenge.] Compute the singular homology groups of the circle $S^1$, directly from the definitions.

**Exercise 3.6.** Compute the singular homology groups of the plane $\mathbb{R}^2$, minus $n$ points. Reference any theorems from Hatcher that you use.

**Exercise 3.7.** Suppose $f_\#\colon C_* \to D_*$ is a *chain map* of chain complexes: a sequence of group homomorphisms so that $\partial f = f\partial$. Show that this induces a well-defined homomorphism $f_*\colon H_*(C) \to H_*(D)$ on homology.

**Exercise 3.8.** [Do not turn in.]

- Suppose that $X$ is a $\Delta$–complex. Let $i\colon \Delta_*(X) \to C_*(X)$ be the inclusion homomorphism. Show that $i$ is a chain map. (Later in the course we will prove $i_*$ is an isomorphism from simplicial homology to singular.)
- Suppose that $f\colon X \to Y$ is a map of topological spaces. Show that the induced function $f_\#\colon C_*(X) \to C_*(Y)$ is a homomorphism and, in fact, a chain map.

**Exercise 3.9.** [Hatcher problem 12, page 132.] Show that chain homotopy of chain maps is an equivalence relation.

**Exercise 3.10.** We say that two chain complexes $C_*$ and $D_*$ are *chain homotopy equivalent* if there are chain maps $f\colon C_* \to D_*$ and $g\colon D_* \to C_*$ so that $g \circ f \sim \mathrm{Id}_C$ and $f \circ g \sim \mathrm{Id}_D$.

Let $C_*$ be the chain complex with $C_1 = C_0 = \mathbb{Z}$, all other chain groups trivial, and with $\partial_1(m) = 2m$. Let $D_*$ be the chain complex with $D_1 = D_0 = \mathbb{Z}^2$, all other chain groups trivial, and with $\partial_1(x, y) = (x - y, x + y)$. Prove that $C_*$ and $D_*$ are chain homotopy equivalent.