Please let me know if any of the problems are unclear or have typos.

Exercise 5.1. [Hard.] Suppose that X is a Δ–complex and suppose that A is a subcomplex. Show that (X, A) is a good pair. (Try doing this first when A is a single vertex. See pages 522-523 of Hatcher for discussion.)

Exercise 5.2. Suppose that $A \subset X$ where X is path-connected and A is non-empty. Show that $H_0(X, A) = 0$, directly from the definitions.

Exercise 5.3. [Do not turn in. Hatcher page 118.] We use $\widetilde{C}_*(X)$ to represent the *reduced* chain complex. That is, set $C_{-1}(X) = \mathbb{Z}$ and replace $\partial_0 = 0$ by the *augmentation map* $\epsilon\colon C_0(X) \to C_{-1}(X)$, where $\epsilon(\sum n_\alpha v_\alpha) = \sum n_\alpha$. If (X, A) is a pair with $A \neq \emptyset$ then show that there is a short exact sequence of chain complexes $0 \to \widetilde{C}_*(A) \xrightarrow{i} \widetilde{C}_*(X) \xrightarrow{q} C_*(X, A) \to 0$. Thus there is an exact triangle of reduced and relative homologies.

Exercise 5.4. [Do not turn in. Hatcher page 118.] Suppose that $B \subset A \subset X$ are subsets; we say (X, A, B) is a *triple*. Show that the inclusion and quotent maps give a short exact sequence of chain complexes $0 \to C_*(A, B) \xrightarrow{i} C_*(X, B) \xrightarrow{q} C_*(X, A) \to 0$. Thus there is an exact triangle of relative homologies.

Exercise 5.5. [Medium. Hatcher page 113, problem 25.] Let $X = [0, 1]$ and set $A = \{0\} \cup \{1/n \mid n \in \mathbb{Z}_+\}$. Note that (X, A) is not a good pair. Show that $H_1(X, A)$ is not isomorphic to $H_1(X/A)$.

Exercise 5.6. [Hatcher page 132 and pages 147-148, problem 11.] Suppose that $A \subset X$ is a subset. Let $i\colon A \to X$ be the inclusion map. Suppose that A is a *retract* of X: that is, there is a map $r\colon X \to A$ with $ri = \mathrm{Id}_A$. Show that there is an isomorphism $H_*(X) \cong H_*(A) \oplus H_*(X, A)$.

Exercise 5.7. [Do not turn in. Hatcher page 119.] Prove that the two versions of excision are equivalent.

Exercise 5.8. After recalling the necessary definitions from Hatcher's proof of excision, verify the following formulas.

- $\partial b + b\partial = 1$.
- $\partial T + T\partial = 1 - S$ and $\partial S = S\partial$.
- $\partial D_k + D_k\partial = 1 - S^k$ and $\partial S^k = S^k\partial$.
- $\partial D + D\partial = 1 - \rho$ and $\partial\rho = \rho\partial$.