Please let me know if any of the problems are unclear or have typos.

Exercise 8.1. [Hatcher page 132, problem 22, and page 140.] Suppose that X is a Δ–complex. Show the following.

- If X has no simplices in dimensions higher than n, then $H_n(X)$ is a free Abelian group.
- The number of n–simplices in X is an upper bound for the size of a minimal generating set for $H_n(X)$.

Let $X^k \subset X$ be the k–skeleton.

- The inclusion map $i\colon X^k \to X$ induces an isomorphism $i_*\colon H_n(X^k) \cong H_n(X)$ when $k > n$.
- The previous statement may fail when $k = n$.

Exercise 8.2. [Medium. Hatcher page 156, problem 16.] Suppose $X = (\Delta^m)^k$ is the k–skeleton of the m–simplex. Compute the reduced homology groups of X.

Exercise 8.3.

- Let M be the Möbius band. Show that $H_1(M) \cong \mathbb{Z}$. Let $\alpha = \partial M$ be the topological boundary of M. Compute the class of α inside of $H_1(M)$.
- Let K_1 be the Klein bottle with a small open disk removed. Show $H_1(K_1) \cong \mathbb{Z}^2$. Let $\alpha = \partial K_1$ be the topological boundary of K_1. Compute the class of α inside of $H_1(K_1)$.

Exercise 8.4. [Medium. Hatcher page 141, example 2.37 does this using cellular homology.] Let $N = N_g$ be the closed non-orientable surface of genus g. That is, $N = \#^g\, \mathbb{RP}^2$ is the connect sum of g copies of $\mathbb{RP}^2$. Compute $H_*(N)$.

Exercise 8.5. [Do not turn in. Hatcher page 137, Propostion 2.33.] Recalling the definitions from Exercise 6.5, if $f\colon X \to X$ is a map then define $Sf\colon SX \to SX$ to be the *suspension* of f: that is, the self-map of SX induced by $f \times \mathrm{Id}$. Prove, when X is a sphere, that $\deg(f) = \deg(Sf)$.

Exercise 8.6. We say a map $f\colon S^n \to S^n$ is *even* if $f(-x) = f(x)$ for all $x \in S^n$. Prove if $f\colon S^n \to S^n$ is even then $\deg(f)$ is even. (You may restrict to the case of $n = 1, 2$.)

Exercise 8.7. [Medium.] Noting $H_2(T^2) \cong \mathbb{Z}$ define the *degree* of a map $f\colon T^2 \to T^2$ to be the degree of the induced map on homology $f_2\colon H_2(T^2) \to H_2(T^2)$. Recall that $T^2 \cong \mathbb{R}^2/\mathbb{Z}^2$. For any two-by-two integer matrix M define $f_M\colon T^2 \to T^2$ via $f_M([x]) = [M(x)]$.

- Make a conjecture about the relationship of $\deg(f_M)$ and M; verify the conjecture for several matrices M.
- Prove your conjecture.