

Please let me know if any of the problems are unclear or have typos.

**Exercise 9.1.** “Let  $p(z) = z^3$  and  $q(z) = z^2$ ; these are polynomials defined on the complex plane. Let  $f(z, t) = (1-t)p(z) + tq(z)$ , for  $t \in [0, 1]$ . Thus  $f$  is a homotopy from  $p$  to  $q$ . Let  $P, Q$ , and  $F$  be the extensions of  $p, q$ , and  $f$  to the one-point compactification  $S^2 = \mathbb{C} \cup \{\infty\}$ . Thus  $\deg(P) = 3$  and  $\deg(Q) = 2$ . Also,  $F$  is a homotopy from  $P$  to  $Q$ . Deduce  $\deg(P) = \deg(Q)$ , and so  $3 = 2$ .”

Verify the true statements and disprove the false ones.

**Exercise 9.2.** Fix  $\{a_k\}_{k=0}^d \subset \mathbb{R}$  with  $a_d \neq 0$ . Define  $p: \mathbb{R}^1 \rightarrow \mathbb{R}^1$  by  $p(x) = \sum a_k x^k$ . Let  $P$  be the extension of  $p$  to the one-point compactification  $S^1 = \mathbb{R}^1 \cup \{\infty\}$ . Check  $P$  is continuous. Compute the degree of  $P$ .

**Exercise 9.3.** Use local degrees to give another proof that the map  $f: S^1 \rightarrow S^1$  defined by  $f(z) = z^d$  has degree equal to  $d$ .

**Exercise 9.4.** Find CW-complex structures for the once-holed torus  $T_1$  and the once-holed Klein bottle  $K_1$ . (These are the torus  $T$  and the Klein bottle  $K$ , minus a small open disk.) Compute the cellular homology groups of each. Show  $T_1$  is not homeomorphic to  $K_1$ .

**Exercise 9.5.** [Hatcher page 156, problem 15.] Suppose  $X$  is a CW-complex. Prove  $H_n^{\text{CW}}(X^n)$  is a free Abelian group.