Please let me know if any of the problems are unclear or have typos.

Exercise 10.1. [Easy.] Show that projective equivalence of points in $\mathbb{R}^{n+1} - \{0\}$ is in fact an equivalence relation.

Exercise 10.2. Suppose P and Q are distinct points in $\mathbb{P}^n$. Show that there is a unique projective line containing them.

Exercise 10.3. Suppose L and M are distinct lines in $\mathbb{P}^2$. Show that $L \cap M$ is a single point. (Thus parallel lines do not exist in projective geometry.)

Exercise 10.4. Suppose U, V, and W are projective subspaces of $\mathbb{P}^n$. Theorem 5.4 tells us that

$$\dim \operatorname{span}(U, V) = \dim U + \dim V - \dim U \cap V$$

where we adopt the convention that $\dim \emptyset = -1$. The principle of inclusion/exclusion suggests the following generalization:

$$\begin{aligned}\dim \operatorname{span}(U, V, W) = &\dim U + \dim V + \dim W \\ &- \dim U \cap V - \dim V \cap W - \dim W \cap U \\ &+ \dim U \cap V \cap W.\end{aligned}$$

Prove this or give a counterexample.

Exercise 10.5. Suppose $p = (1 : 0)$, $q = (0 : 1)$, and $r = (1 : 1)$ are three points in $\mathbb{P}^1$. Find all six elements of $\mathrm{PGL}(2, \mathbb{R})$ that preserve the set $\{p, q, r\}$.