Please let me know if any of the problems are unclear or have typos.

Exercise 3.1. Choose P to be one of the tetrahedron, cube, octahedron, dodecahedron, or icosahedron. For your choice of P, carry out the following exercise.

Show that after scaling appropriately P can be inscribed in S^2, the unit sphere. This done, compute the coordinates of the vertices of P. Compute the lengths of the edges of P. Finally, compute the *dihedral angle* of P: the angle made by a pair of adjacent faces of P, when intersected with a plane orthogonal to both.

Exercise 3.2. Fix $b \in \mathbb{R}^n$. Define $\mathrm{Trans}_b \colon \mathbb{R}^n \to \mathbb{R}^n$ by $\mathrm{Trans}_b(x) = x + b$. Prove Trans_b is an isometry. Show that Trans_b preserves angles.

Exercise 3.3. Suppose that $A \in O(n)$ is an orthogonal matrix. Define $\mathrm{Ortho}_A \colon \mathbb{R}^n \to \mathbb{R}^n$ by $\mathrm{Ortho}_A(x) = Ax$. Prove Ortho_A is an isometry. Show that the composition of isometries is again an isometry. Deduce that $\mathrm{Trans}_b \circ \mathrm{Ortho}_A$, taking x to $Ax + b$, is an isometry. (This is the converse of the statement made in class.)

Exercise 3.4. With Trans_b as defined in Exercise 3.2, show that the "translation subgroup" is normal in $\mathrm{Isom}(\mathbb{R}^n)$. That is, for any $c \in \mathbb{R}^n$ and for any $T \in \mathrm{Isom}(\mathbb{R}^n)$ there is a vector $c' \in \mathbb{R}^n$ so that

$$\mathrm{Trans}_{c'} = T \circ \mathrm{Trans}_c \circ T^{-1}.$$

Verify this and find c'.

Exercise 3.5. For each of the following isometries $T \in \mathrm{Isom}(\mathbb{R}^2)$ find an orthogonal matrix $A \in O(2)$ and a vector $b \in \mathbb{R}^2$ so that $T(x) = Ax + b$.

- Reflection in the line containing the points $(1, 0)$ and $(0, 2)$.
- Reflection in the line containing the points $(1, 0)$ and $(0, 2)$ followed by reflection in the line containing the points $(1, 0)$ and $(3, 1)$.
- Rotation by $\pi/6$ (anti-clockwise) about the point $(1, 0)$.
- Translation by $(1, 0)$ followed by a rotation by $\pi/4$ about the origin.