Please let me know if any of the problems are unclear or have typos.

Exercise 5.1. Exercise 2.6 from the book, regarding halfturns.

Exercise 5.2. Suppose that $A \in O(n+1)$ is an orthogonal matrix. We may consider A as a function from $\mathbb{R}^{n+1}$ to itself; thus we can define $A|S^n$ to be the restriction of A to S^n. Show that $A|S^n$ is an isometry of S^n, equipped with the spherical metric.

Exercise 5.3. Suppose that $p, q \in S^n$. Show via a direct computation that

$$2 \arcsin\left(\frac{|p-q|}{2}\right) = \arccos(p \cdot q).$$

Thus the two definitions of distance in S^n, given in class, agree.

Exercise 5.4. [A version of Exercise 3.3, from the book.] Suppose that p and q are distinct points in the metric space X. Define $B(p,q) = \{x \in X \mid d_X(x,p) = d_X(x,q)\}$. This is the set of points *equidistant* from p and q. Show the following.

- If $X = \mathbb{R}^2$ with the usual metric, then $B(p,q)$ is a line.
- If $X = S^2$ with the usual metric, then $B(p,q)$ is a great circle.

Exercise 5.5. [A version of Exercise 3.6, from the book.] Suppose that ΔPQR is a spherical triangle. Let A, B, C be the sidelengths opposite P, Q, R respectively. Let α, β, γ be the internal angles adjacent to P, Q, R respectively. Prove the spherical sine law:

$$\frac{\sin\alpha}{\sin A} = \frac{\sin\beta}{\sin B} = \frac{\sin\gamma}{\sin C}.$$