

Please let me know if any of the problems are unclear or have typos.

Exercise 6.1. For the purposes of this problem we assume that the surface of the Earth is a perfect sphere, with radius 6378 km. Consider the three cities Portland, USA; Qassimiut, Greenland; and Reynosa, Mexico. According to Google these are located at $(45.52^\circ N, 122.68^\circ W)$; $(60.78^\circ N, 47.16^\circ W)$; and $(26.09^\circ N, 98.28^\circ W)$ respectively.

- For each of the cities convert the given spherical coordinates to rectangular coordinates.
- For each pair of cities, compute the great circle distance between them.
- For each triple of cities, compute the angle formed by the first and third as viewed from the second.

As a sanity check, note that Greenwich Observatory is at $(51.48^\circ N, 0^\circ W)$, again according to Google. In rectangular coordinates this is approximately $(3972, 0, 4990)$. Thus the distance between Greenwich and the equator is about 5731 km; the distance between Greenwich and Portland is about 7951 km. (Suggestion: write a computer program to do all calculations; check your work against on-line sources. Attach a print-out of your code.)

Exercise 6.2. [Hard.] Suppose that $\Omega \subset S^2$ is a topologically a disk, bounded by n arcs of great circles. Give a formula for the area of Ω in terms of its internal angles $\{\alpha_i\}_{i=1}^n$. Carefully justify all steps of your argument.

Exercise 6.3. Suppose $\Delta \subset S^2$ is a spherical triangle. We call Δ *equiangular* if the three angles of Δ are equal. We call Δ *Platonic* if copies of Δ tile the sphere (that is, vertices only meet vertices, edges only meet edges, and interiors are disjoint). Find all equiangular Platonic triangles and prove your list is complete.

Exercise 6.4.

- Describe all of the isometries of \mathbb{E}^2 that can be obtained as the composition of exactly two reflections. Briefly discuss *uniqueness*: that is, if T is a composition of a pair of reflections, then in how many ways is T a composition of a pair of reflections?
- Do the same for isometries of S^2 . Which ones are obtained as a composition of a pair of reflections, and in how many ways?