

Please let me know if any of the problems are unclear or have typos.

Exercise 4.1. Show that the map $p: \mathbb{R} \rightarrow S^1$ defined by $p(t) = \exp(2\pi it)$ is a covering map.

Exercise 4.2. With notation as set in class: check the following claims, needed in the proof that Φ is a homomorphism.

- Show that $\tilde{\omega}_{m+n} \stackrel{\partial}{\simeq} \tilde{\omega}_m * (\tau_m \circ \tilde{\omega}_n)$.
- Suppose that $\alpha \stackrel{\partial}{\simeq} \beta$ are paths in \mathbb{R} . Show that $p \circ \alpha \stackrel{\partial}{\simeq} p \circ \beta$ as paths in S^1 .
- Suppose that α, β are paths in \mathbb{R} with $\alpha(1) = \beta(0)$. Show that $p \circ (\alpha * \beta) = (p \circ \alpha) * (p \circ \beta)$.

Exercise 4.3. Let F be the *figure eight graph*: the graph with one vertex and two edges. List all connected two- and three-fold covers of F , up to isomorphism. Give an argument that your lists are complete.

Exercise 4.4. [Hard] Let $T = S^1 \times S^1$ be the torus. For each $d > 0$, count the isomorphism classes of connected d -fold covers of T .

Exercise 4.5. Problem 6, page 38, from Hatcher's book.