Please let me know if any of the problems are unclear or have typos.

Exercise 7.1. [Do not turn in.] Suppose X is a topological space. We define CX to be the *cone* on X: that is

$$CX = X \times I \Big/ (x,0) \sim (y,0) \text{ for all } x, y \in X.$$

The point $a = [(x,0)]$ is called the *apex* of the cone. Show that the cone CX deformation retracts to its apex. Deduce $\pi_1(CX, a)$ is trivial.

Exercise 7.2. Suppose G, H are nontrivial groups. Show that the free product $G * H$ is not isomorphic to $\mathbb{Z}^2$.

Exercise 7.3. Suppose that $\{G_\alpha\}$ is a countable collection of countable groups. Show that $*_\alpha G_\alpha$ is countable.

For the next two problems we need the following definition. Let $C_n \subset \mathbb{R}^2$ be the circle of radius $1/n$ centered at $(1/n, 0) \in \mathbb{R}^2$. We define $H \subset \mathbb{R}^2$, the *Hawaiian earring*, to be the union $H = \cup_{n=1}^\infty C_n$. We take H to be a pointed space, with basepoint at $h = (0,0)$. Let $\Gamma = \pi_1(H, h)$.

Exercise 7.4.

- For all $n > 0$ give a retraction $r_n \colon H \to C_n$. Explain why r_n is continuous.
- Show that $\Gamma = \pi_1(H, h)$ is uncountable. Briefly explain why Γ is not isomorphic to

$$\pi_1\left(\bigvee_{n\in\mathbb{N}} S^1\right) \cong \mathop{*}_{n\in\mathbb{N}} \mathbb{Z}.$$

Exercise 7.5.

- Show that $H \cong H \vee H$. (Recall that we use $h = (0,0)$ as the basepoint.)
- [Medium.] Show that the homeomorphism above does not induce an isomorphism between Γ and $\Gamma * \Gamma$.