

Please let me (Saul) know if any of the problems are unclear or have typos. Please turn in a single worked exercise — write your name, the date, and the problem you are solving at the top of the page. If you collaborate with other students, please also include their names.

**Exercise 2.1.** [Medium. Hatcher page 104 and page 522.] Suppose that  $X$  is a topological space, equipped with a  $\Delta$ -complex structure  $\{\sigma_\alpha: \Delta_\alpha \rightarrow X\}_{\alpha \in I}$ . Prove that  $X$  is Hausdorff.

**Exercise 2.2.** [Medium. Hatcher page 130 and page 520.] Suppose that  $X$  is a topological space, equipped with a  $\Delta$ -complex structure  $\{\sigma_\alpha: \Delta_\alpha \rightarrow X\}_{\alpha \in I}$ . Suppose that  $K \subset X$  is compact. Prove that  $K$  meets only finitely many open simplices.

**Exercise 2.3.** Show that every compact, connected, orientable surface without boundary admits a  $\Delta$ -complex structure.

**Exercise 2.4.** [Medium.] Suppose that  $X$  and  $Y$  are equipped with  $\Delta$ -complex structures. Show that  $X \times Y$  admits a  $\Delta$ -complex structure.

**Exercise 2.5.** Show that  $\mathbb{Q}$  is not isomorphic to a free Abelian group.

**Exercise 2.6.** [Hatcher's extra problems, 2.1.1.] Let  $X$  be the circle, equipped with the  $\Delta$ -complex structure with  $n$  vertices and  $n$  edges. Compute the simplicial homology of  $X$ , directly from the definitions.

**Exercise 2.7.** List all  $\Delta$ -complexes that can be made from a single two-simplex. (I believe there are seven.) For each, compute the simplicial homology groups.

**Exercise 2.8.** [Hard.] Compute the simplicial homology groups of  $\Delta^n$ , the  $n$ -simplex equipped with the natural  $\Delta$ -complex structure.

**Exercise 2.9.** [Hard.] Let  $C(r)$  be the circle in the plane centered at  $(r, 0)$  and with radius  $r$ . Define  $H = \cup_{n \in \mathbb{Z}_+} C(1/n)$  and endow it with the subspace topology. This is the *Hawaiian earring*. Show that  $H$  does not admit a  $\Delta$ -complex structure.