Please let me know if any of the problems are unclear or have typos. Please turn in a *single* worked exercise. Record your name, the date, and the problem solved. Please also write the names of people and/or references you consult.

Exercise 9.1. [Borsuk] Show that the following are equivalent.

1. For any map $f\colon S^n \to \mathbb{R}^n$ there is a point $x \in S^n$ so that $f(-x) = f(x)$.
2. For any odd map $g\colon S^n \to \mathbb{R}^n$ there is a point $x \in S^n$ so that $g(x) = 0$.
3. There is no odd map $h\colon S^n \to S^{n-1}$.
4. There is no map $k\colon B^n \to S^{n-1}$ so that $k|S^{n-1}$ is odd.

Now deduce one (and so all) of these from the "odd theorem": an odd map $f\colon S^n \to S^n$ has odd degree.

Exercise 9.2. Set $C = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = z^2\}$. For all points $p \in C$ compute the local homology groups of C at p.

Exercise 9.3. Show that S^n, P^n, and T^n — the sphere, projective space, and torus — are manifolds.

Exercise 9.4. Show that S^n and T^n are orientable. Show that P^n is orientable if and only if n is odd.

Exercise 9.5. Use local degrees to give another proof that the map $f\colon S^1 \to S^1$ defined by $f(z) = z^d$ has degree equal to d.

Exercise 9.6. Find CW–complex structures for the once-holed torus T_1 and the once-holed Klein bottle K_1. (These are the torus T and the Klein bottle K, minus a small open disk.) Compute the cellular homology groups of each. Show T_1 is not homeomorphic to K_1.

Exercise 9.7. [Hatcher page 156, problem 15.] Suppose X is a CW-complex. Prove $H_n^{\mathrm{CW}}(X^n)$ is a free Abelian group.