The required problems are Exercises 2.1, 2.2, and 2.6. Please let me know if any of the problems are unclear or have typos.

Exercise 2.1. Give an example of an isometric embedding $f\colon X \to X$ which is not surjective.

Exercise 2.2. [Medium] Give an example of a metric space (X, d) which does not isometrically embed into the euclidean plane $\mathbb{E}^2$: that is, into $\mathbb{R}^2$ with the usual metric.

Exercise 2.3. State the definition of arccos. Now suppose $u, v \in \mathbb{R}^n$ are vectors. We define the angle between u and v to be $\theta_{u,v} = \arccos(u \cdot v/|u||v|)$. Show that $\theta_{u,v}$ is well-defined. Now compute the angles between the following pairs of vectors in $\mathbb{R}^2$.

- $(1, 0)$ and $(0, 1)$.
- $(0, 1)$ and $(-1/2, \sqrt{3}/2)$.
- $(-1/2, \sqrt{3}/2)$ and $(-1, 1)$.
- $(-1, 1)$ and $(-1, 0)$.

Exercise 2.4. For $a, b \in \mathbb{R}^n$ we define

$$[a, b] = \left\{ c \in \mathbb{R}^n \;\middle|\; \exists t \in [0, 1] \text{ so that } c = (1-t)a + tb \right\}.$$

Suppose $x, y, z \in \mathbb{R}^n$ satisfy $x \in [y, z]$, $y \in [z, x]$, and $z \in [x, y]$. What can you deduce? Justify your answer.

Exercise 2.5. Suppose (X, d_X) and (Y, d_Y) are metric spaces. An *isometry* $f\colon X \to Y$ is an isometric embedding that is, additionally, surjective. Let $\mathrm{Isom}(X)$ be the set of isometries from X to itself. Show $\mathrm{Isom}(X)$ is a group, if we take the group operation to be function composition.

Exercise 2.6. Set $X = [-1, 1]$ and, for $x, y \in X$ define $d(x, y) = |x - y|$. Verify this is a metric. Find the group $\mathrm{Isom}(X)$ and justify your answer.

Exercise 2.7. [Exploration] Fix $n \geq 2$. For $p \geq 1$ and for $x, y \in \mathbb{R}^n$ define

$$d^p(x, y) = \left(\sum_i |x_i - y_i|^p \right)^{1/p}.$$

Verify that $(\mathbb{R}^n, d^p)$ is a metric space. Show that $(\mathbb{R}^n, d^p)$ is isometric to $(\mathbb{R}^n, d^q)$ if and only if $p = q$.